Lecture 14

Topics in Development Economics: Growth Theories: the Solow Model

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Contents

Motivation

- Average real incomes today in the United States and Western Europe are between 10 and 30 times larger than a century ago;
- Average real incomes today in the United States and Western Europe are between 50 and 300 times larger than two centuries ago.
- Growth has been rising over most of modern history.

Motivation

- There are also enormous differences in standards of living across parts of the world.
  - Average real incomes in the United States, Germany, and Japan exceed those in Bangladesh and Kenya by a factor of 20.
- The most striking examples of large changes in relative incomes are growth miracles and growth disasters.

Motivation

Growth miracles

episodes where growth in a country far exceeds the world average over an extended period, with the result that the country moves rapidly up the world income distribution.

Examples:

- Japan from the end of World War II to 1990;
- The newly industrializing countries (NICs) of East Asia starting 1960:
  - South Korea;
  - Taiwan;
  - Singapore;
  - Hong Kong.
- China starting 1980.

Motivation

Growth disasters

episodes where a country's growth falls far short of the world average.

Examples:

- Argentina;
- Sub-Saharan African countries:
  - Chad;
  - Ghana;
  - Mozambique.

Inputs and Output

The production function takes the form

$$Y(t) = F(K(t), A(t)L(t)) \tag{1.1}$$

where

Y=output;
K=capital;
L=labor;
A="knowledge" or the "effectiveness of labor".

- A and L enter multiplicatively.
- AL is called effective labor, and technological progress that enters in this fashion is known as labor-augmenting or Harrod-neutral.

Assumptions

Assumptions Concerning the Production Function

- Production function has <u>constant returns to scale</u> in its two arguments:
  - Doubling the quantities of capital and effective labor doubles the amount produced.
  - $F(cK, cAL) = cF(K, AL) \quad \text{for all } c \geq 0 \qquad (1.2)$
- The assumption of constant returns is a combination of two separate assumptions:
  1. The economy is big enough that the gains from specialization have been exhausted.
  2. Inputs other than capital, labor, and knowledge are relatively unimportant.
     - The model neglects land and other natural resources.

Assumptions

Assumptions Concerning the Production Function

1. The economy is big enough that the gains from specialization have been exhausted.
    1. *What happens if the economy is very small?*
2. Inputs other than capital, labor, and knowledge are relatively unimportant.
    2. *If natural resources are important, what happens if we double capital and labor?*

Assumptions

Assumptions Concerning the Production Function

1. The economy is big enough that the gains from specialization have been exhausted.
   1. *What happens if the economy is very small?*
      - In a very small economy, there are likely to be enough possibilities for further specialization that doubling the amounts of capital and labor more than doubles output.
2. Inputs other than capital, labor, and knowledge are relatively unimportant.
   2. *If natural resources are important, what happens if we double capital and labor?*
      - If natural resources are important, doubling capital and labor could less than double output.

Assumptions

Assumptions Concerning the Production Function

The assumption of constant returns allows us to work with the production function in *intensive form*.

Setting $c = 1/AL$ yields

$$F(\frac{K}{AL}, 1) = \frac{1}{AL}F(K, AL) \tag{1.3}$$

$\frac{K}{AL}$ is the amount of capital per unit of effective labor;
$\frac{F(K,AL)}{AL}$ is $\frac{Y}{AL}$, output per unit of effective labor.

Assumptions Concerning the Production Function

Define:

$k = K/AL$,
$y = Y/AL$,
$f(k) = F(k, 1)$

Then we can rewrite output per unit of effective labor as a function of capital per unit of effective labor:

$$y = f(k) \tag{1.4}$$

Assumptions

Assumptions Concerning the Production Function

The intensive-form production function, $f(k)$ is assumed to satisfy:

- $f(0) = 0$,
- $f'(k) > 0$,
- $f''(k) < 0$

This implies that:

- The marginal product of capital is positive;
- The marginal product of capital declines as capital (per unit of effective labor) rises.

Assumptions

Assumptions Concerning the Production Function

In addition, $f(.)$ is assumed to satisfy the *Inada conditions*:

- $\lim_{k \to 0} f'(k) = \infty$
- $\lim_{k \to \infty} f'(k) = 0$

These conditions state that

- The marginal product of capital is very large when the capital stock is sufficiently small;
- The marginal product of capital becomes very small as the capital stock becomes large.

Assumptions Concerning the Production Function

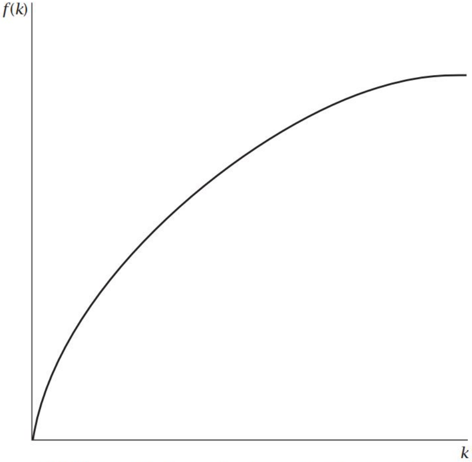

FIGURE 1.1 An example of a production function

Assumptions Concerning the Production Function

A specific example of a production function is the *Cobb-Douglas* function:

$$F(K, AL) = K^{\alpha}(AL)^{1-\alpha}, \qquad 0 < \alpha < 1 \tag{1.5}$$

3. *Check that it satisfies the condition of constant returns to scale.*

Assumptions Concerning the Production Function

A specific example of a production function is the *Cobb-Douglas* function:

$$F(K, AL) = K^{\alpha}(AL)^{1-\alpha}, \qquad 0 < \alpha < 1 \tag{1.5}$$

3. *Check that it satisfies the condition of constant returns to scale.*

$$\begin{aligned} F(cK, cAL) &= (cK)^{\alpha}(cAL)^{1-\alpha} \\ &= c^{\alpha}c^{1-\alpha}K^{\alpha}(AL)^{1-\alpha} \\ &= cF(K, AL) \end{aligned} \tag{1.6}$$

Assumptions Concerning the Production Function

To find the intensive form of the production function, divide both inputs by AL:

$$
\begin{aligned}
f(k) &\equiv F(\frac{K}{AL}, 1) \\
&= (\frac{K}{AL})^{\alpha} \\
&= k^{\alpha}
\end{aligned}
\tag{1.7}
$$

4. *Check that it satisfies the following conditions:*
   - $f(0) = 0$,
   - $f'(k) > 0$,
   - $f''(k) < 0$
5. *Check that it satisfies the Inada conditions:*
   - $\lim_{k \to 0} f'(k) = \infty$
   - $\lim_{k \to \infty} f'(k) = 0$

The Evolution of Inputs into Production

- The model is set in continuous time.
    6. *What does this mean?*
- The initial levels of capital, labor, and knowledge are taken as given, and are assumed to be strictly positive.
- Labor and knowledge grow at constant rates:

$$\dot{L}(t) = nL(t) \tag{1.8}$$

$$\dot{A}(t) = gA(t) \tag{1.9}$$

n, g are exogenous parameters;
dot over a variable denotes a derivative with respect to time (*Example:* $\dot{X}(t) = \frac{dX(t)}{dt}$)

The Evolution of Inputs into Production

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  6. *What does this mean?*
     - The variables of the model are defined at every point in time.
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The Evolution of Inputs into Production

Growth rate

of a variable refers to its proportional rate of change.

Example: The growth rate of X refers to the quantity $\frac{\dot{X}(t)}{X(t)}$.

- (1.8) implies that the growth rate of L is constant and equal to n.
- (1.9) implies that A's growth rate is constant and equal to g.

Assumptions

The Evolution of Inputs into Production

Growth rate of a variable equals the rate of change of its natural log:

- $\frac{\dot{X}(t)}{X(t)} = \frac{dlnX(t)}{d(t)}$

$$\begin{aligned} \frac{dlnX(t)}{dt} &= \frac{dlnX(t)}{dX(t)} \frac{dX(t)}{dt} \\ &= \frac{1}{X(t)} \dot{X}(t) \end{aligned} \tag{1.10}$$

Assumptions

The Evolution of Inputs into Production

Applying the earlier result to (1.8) and (1.9) we get:

$$lnL(t) = [lnL(0)] + nt \tag{1.11}$$

$$lnA(t) = [lnA(0)] + gt \tag{1.12}$$

$L(0), A(0)$ are the values of L and A at time 0.

Exponentiating both sides of these equations gives us

$$L(t) = L(0)e^{nt} \tag{1.13}$$

$$A(t) = A(0)e^{gt} \tag{1.14}$$

Thus, our assumption is that L and A each grow exponentially.

The Evolution of Inputs into Production

- Output is divided between consumption and investment.
- The fraction of output devoted to investment, s, is exogenous and constant.
- One unit of output devoted to investment yields one unit of new capital.
- Existing capital depreciates at rate δ.

$$\dot{K}(t) = sY(t) - \delta K(t) \tag{1.15}$$

The Evolution of Inputs into Production

The Solow model is very much simplified in a variety of ways:

- There is only a single good;
- Government is absent;
- Fluctuations in employment are ignored;
- Production is described by an aggregate production function with just three inputs;
- The rates of saving, depreciation, population growth, and technological progress are constant.

$\Rightarrow$ *Does this mean that it is a bad model because it is not realistic?*

The Evolution of Inputs into Production

$\Rightarrow$ *Does this mean that it is a bad model because it is not realistic?*

Not necessarily!

- The purpose of a model is not to be realistic.
- A model's purpose is to provide insights about particular features of the world.
- If a simplifying assumption causes a model to give incorrect answers *to the questions it is being used to address*, then the lack of realism may be a defect.
- If the simplification does not cause the model to provide incorrect answers to the questions it is being used to address, then the lack of realism is a virtue:
  - By isolating the effect of interest more clearly, the simplification makes it easier to understand.

The Dynamics of k

- We would like to determine the behavior of the economy we have just described.
- The evolution of two of the three inputs into production, labor and knowledge, is exogenous.
- Thus to characterize the behavior of the economy, we must analyze the behavior of the third input, capital.

The Dynamics of k

Since $k = \frac{K}{AL}$, we can use the chain rule to find

$$\dot{k}(t) = \frac{\dot{K}(t)}{A(t)L(t)} - \frac{K(t)}{[A(t)L(t)]^2}[A(t)\dot{L}(t) + L(t)\dot{A}(t)]$$
$$= \frac{\dot{K}(t)}{A(t)L(t)} - \frac{K(t)}{A(t)L(t)}\frac{\dot{L}(t)}{L(t)} - \frac{K(t)}{A(t)L(t)}\frac{\dot{A}(t)}{A(t)} \quad (1.16)$$

Substituting leads to

$$\dot{k}(t) = \frac{sY(t) - \delta K(t)}{A(t)L(t)} - k(t)n - k(t)g$$
$$= s\frac{Y(t)}{A(t)L(t)} - \delta k(t) - nk(t) - gk(t) \quad (1.17)$$

The Dynamics of k

Finally, using the fact that $\frac{Y}{AL}$ is given by $f(k)$, we have

$$\dot{k}(t) = sf(k(t)) - (n + g + \delta)k(t) \tag{1.18}$$

This is the key equation of the Solow model. It states that the rate of change of the capital stock per unit of effective labor is the difference between two terms:

1. $sf(k)$ is actual investment per unit of effective labor:
    - output per unit of effective labor is $f(k)$;
    - fraction of that output that is invested is s.
2. $(n + g + \delta)k$ is **break-even investment**, the amount of investment that must be done just to keep k at its existing level.

The Dynamics of k

There are two reasons that some investment is needed to prevent k from falling.

1. Existing capital is depreciating. This capital must be replaced to keep the capital stock from falling.
    - This is the δk term.
2. The quantity of effective labor is growing. Thus doing enough investment to keep the capital stock (K) constant is not enough to keep the capital stock per unit of effective labor (k) constant.
    - Since the quantity of effective labor is growing at rate $n + g$, the capital stock must grow at rate $n + g$ to hold k steady.
    - This is the $(n + g)$ term.

The Dynamics of the Model

The Dynamics of k

- When actual investment per unit of effective labor exceeds the investment needed to break even, k is *rising*.
- When actual investment falls short of break-even investment, k is *falling*.
- When the two are equal, k is *constant*.

The Dynamics of k

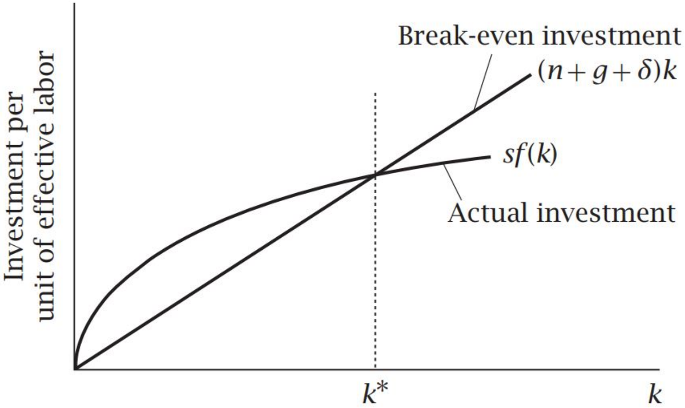


FIGURE 1.2 Actual and break-even investment

The Dynamics of k

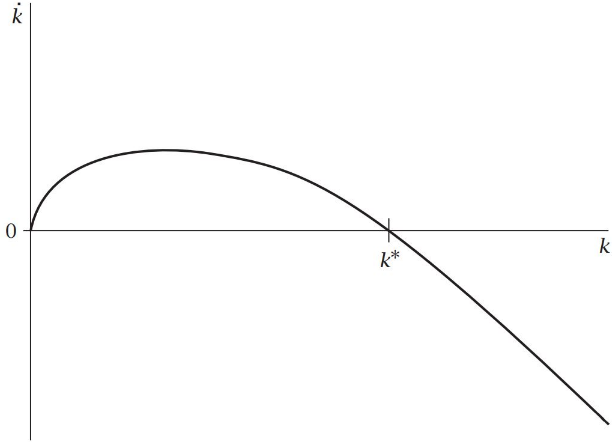


FIGURE 1.3 The phase diagram for *k* in the Solow model

The Dynamics of the Model

The Balanced Growth Path

How do the variables behave when k equals k^*?

- By assumption, labor and knowledge are growing at rates n and g, respectively.
- The capital stock, K, equals ALk.
- Since k is constant at k^*, K is growing at rate $n + g$.
- With both capital and effective labor growing at rate $n + g$, the assumption of constant returns implies that output, Y, is also growing at that rate.
- Finally, capital per worker, K/L, and output per worker, Y/L, are growing at rate g.

The Balanced Growth Path

Thus the Solow model implies that, regardless of its starting point, the economy converges to a balanced growth path - a situation where each variable of the model is growing at a constant rate.

On the balanced growth path, the growth rate of output per worker is determined solely by the rate of technological progress.

The Impact of a Change in the Saving Rate

- Consider a Solow economy that is on a balanced growth path.
- Suppose that there is a permanent increase in s.

The Impact on Output

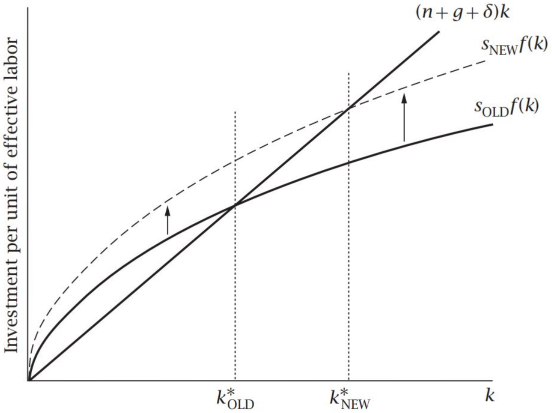
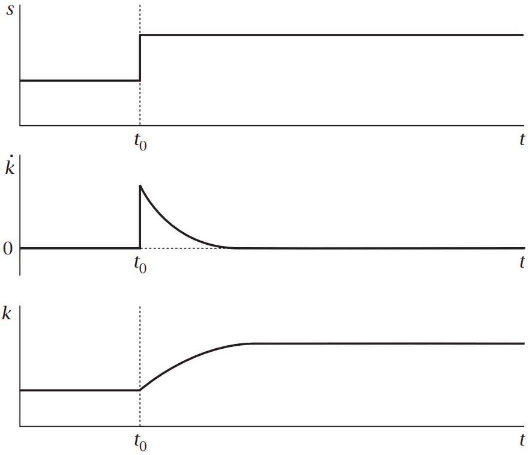


FIGURE 1.4 The effects of an increase in the saving rate on investment

The Impact of a Change in the Saving Rate

The Impact on Output



The Impact of a Change in the Saving Rate

The Impact on Output

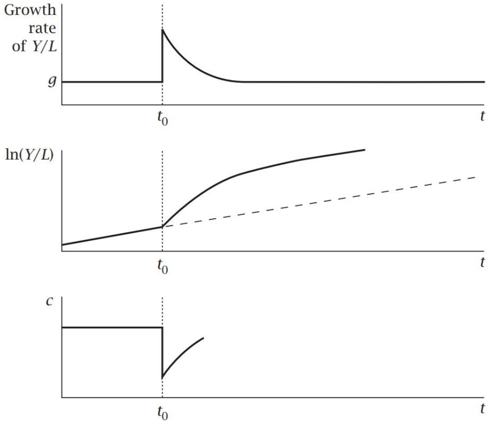


FIGURE 1.5 The effects of an increase in the saving rate

The Impact on Output

In sum, a change in the saving rate has a *level effect* but not a *growth effect* :

- It changes the economy's balanced growth path, and thus the level of output per worker at any point in time;
- But it does not affect the growth rate of output per worker on the balanced growth path.

In general, in the Solow model

- Only changes in the rate of technological progress have growth effects.
- All other changes have only level effects.

The Impact of a Change in the Saving Rate

The Impact on Consumption

- Consumption per unit of effective labor equals output per unit of effective labor, $f(k)$, times the fraction of that output that is consumed, $1 - s$.
- Thus, since s changes discontinuously at t_0 and k does not, initially consumption per unit of effective labor jumps downward.
- Consumption then rises gradually as k rises and s remains at its higher level.
- Whether consumption eventually exceeds its level before the rise in s is not immediately clear.

The Impact of a Change in the Saving Rate

The Impact on Consumption

- Let c^* denote consumption per unit of effective labor on the balanced growth path.

$$c^* = f(k^*) - (n + g + \delta)k^* \tag{1.19}$$

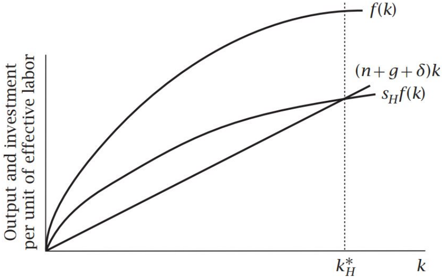
  Then,

$$\frac{\partial c^*}{\partial s} = [f'(k^*(s, n, g, \delta)) - (n + g + \delta)] \frac{\partial k^*(s, n, g, \delta)}{\partial s} \tag{1.20}$$

- The increase in s raises k^*:
  - $\frac{\partial k^*}{\partial s} > 0$
- Thus, whether the increase raises or lowers consumption in the long run depends on whether
  - $f'(k^*)$ - the marginal product of capital

    $><$
  - $n + g + \delta$

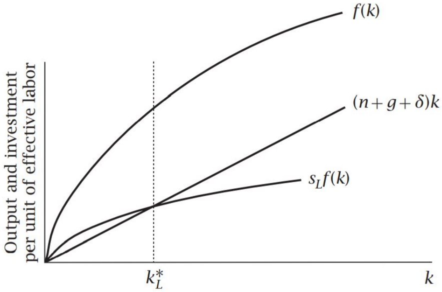
The Impact on Consumption



- Here s is high, and so k^* is high;
- $f'(k^*)$ is less than $n + g + \delta$.

  $\Rightarrow$ An increase in the saving rate lowers consumption.

The Impact on Consumption

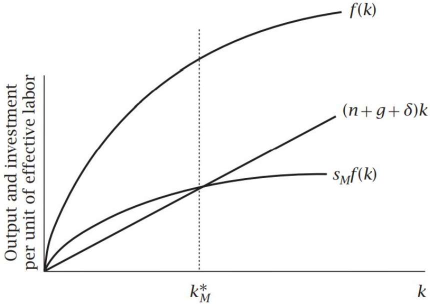


- Here s is low, and so k^* is low;
- $f'(k^*)$ is greater than $n+g+\delta$.

$\Rightarrow$ An increase in the saving rate raises consumption.

The Impact of a Change in the Saving Rate

The Impact on Consumption



- Here s is at the level that causes $f'(k^*)$ to just equal $n+g+\delta$.
- $f(k)$ and $(n+g+\delta)k$ loci are parallel at $k=k^*$.

$\Rightarrow$ An increase in the saving rate has no effect on consumption.

$\Rightarrow$ Consumption is at its maximum possible level among balanced growth paths.

$\Rightarrow$ This value of k^* is known as the golden-rule level of the capital stock.

Quantitative Implications

The Effect on Output in the Long Run

The long-run effect of a rise in saving on output is given by

$$\frac{\partial y^*}{\partial s} = f'(k^*)\frac{\partial k^*(s,n,g,\delta)}{\partial s} \tag{1.21}$$

$y^* = f(k^*)$ is the level of output per unit of effective labor on the balanced growth path.

After a series of derivations, we get

$$\frac{s}{y^*}\frac{\partial y^*}{\partial s} = \frac{\alpha_K(k^*)}{1-\alpha_K(k^*)} \tag{1.27}$$

This is the elasticity of the balanced-growth-path level of output with respect to the saving rate.

where $\alpha_K(k^*)$ is the elasticity of output with respect to capital at $k = k^*$.

The Effect on Output in the Long Run

- In most countries, the share of income paid to capital is about one-third.
- Then, the elasticity of output with respect to the saving rate in the long run is about one-half.
  - *Example:* A 10% increase in the saving rate (from 20% of output to 22%, for instance) raises output per worker in the long run by 5%.
  - *Example:* Even a 50% increase in s raises y^* only by about 22%.

  $\Rightarrow$ Significant changes in saving have only moderate effects on the level of output on the balanced growth path.

The Speed of Convergence

Our goal is to determine how rapidly k approaches k^*.

After a series of derivations we get:

$$\lambda \equiv -\frac{\partial \dot{k}(k)}{\partial k}|_{k=k^*} = [1 - \alpha_K(k^*)](n + g + \delta) \qquad (1.31)$$

We can calibrate (1.31) to see how quickly actual economies are likely to approach their balanced growth paths.

The Speed of Convergence

- Typically, $n + g + \delta$ is 6% per year:
  - 1 to 2% population growth;
  - 1 to 2% growth in output per worker;
  - 3 to 4% depreciation.
- If capital's share is roughly one-third, $(1 - \alpha_k)(n + g + \delta)$ is roughly 4%.

  $\Rightarrow$ k and y move 4% of the remaining distance toward k^* and y^* each year, and take approximately 17 years to get halfway to their balanced-growth-path values.

The Speed of Convergence

- So if there is a 10% increase in the saving rate,
  - output is 0.04(5%)=0.2% above its previous path after 1 year;
  - 0.5(5%)=2.5% above after 17 years;
  - asymptotically approaches 5% above the previous path.

$\Rightarrow$ Not only is the overall impact of a substantial change in the saving rate modest, but it does not occur very quickly.

The Solow Model and the Central Questions of Growth Theory

The Solow model identifies two possible sources of variation in output per worker:

1. differences in capital per worker (K/L);
2. differences in the effectiveness of labor (A).

- But only growth in the effectiveness of labor can lead to permanent growth in output per worker.
- The impact of changes in capital per worker on output per worker is modest.

  $\Rightarrow$ Only differences in the effectiveness of labor have any reasonable hope of accounting for the vast differences in wealth across time and space.

The Solow Model and the Central Questions of Growth Theory

- The central conclusion of the Solow model is that the variations in the accumulation of physical capital do not account for a significant part of either worldwide economic growth or cross-country income differences.
- As for effectiveness of labor, unfortunately, the Solow model has little to say about it.
  - The growth of the effectiveness of labor is exogenous:
    - The model takes as given the behavior of the variable that it identifies as the driving force of growth.
    - In a way, we have been modeling growth by assuming it!

The Solow Model and the Central Questions of Growth Theory

And what is effectiveness of labor?

- The model does not identify what it is:
    - It is just factors other than labor and capital that affect output.
    - Thus saying that differences in income are due to differences in the effectiveness of labor is no different than saying that they are not due to differences in capital per worker.
- It could be:
    - abstract knowledge;
    - the education and skills of the labor force;
    - the strength of property rights;
    - the quality of infrastructure;
    - cultural attitudes toward entrepreneurship and work, and so on.

Convergence

An issue that has attracted considerable attention in empirical work on growth is whether poor countries tend to grow faster than rich countries.

Convergence

There are at least three reasons that one might expect such convergence.

1. The Solow model predicts that countries converge to their balanced growth paths.
   - Thus to the extent that differences in output per worker arise from countries being at different points relative to their balanced growth paths, one would expect poor countries to catch up to rich ones.
2. The Solow model implies that the rate of return on capital is lower in countries with more capital per worker.
   - Thus there are incentives for capital to flow from rich to poor countries; this will also tend to cause convergence.
3. If there are lags in the diffusion of knowledge, income differences can arise because some countries are not yet employing the best available technologies.
   - These differences might tend to shrink as poorer countries gain access to state-of-the-art methods.

Convergence

Baumol (1986) examines convergence from 1870 to 1979 among the 16 industrialized countries. Baumol regresses output growth over this period on a constant and initial income.

$$ln[(\frac{Y}{N})_{i,1979}] - ln[(\frac{Y}{N})_{i,1870}] = a + b\ ln[(\frac{Y}{N})_{i,1870}] + \epsilon_i \qquad (1.36)$$

$ln(Y/N)$ is log income per person;
ϵ is an error term;
i indexes countries.

Convergence

7. *If there is convergence, what would be the sign of b?*

Convergence

7. *If there is convergence, what would be the sign of b?*
   - If there is convergence, b will be negative:
     - Countries with higher initial incomes have lower growth.
   - A value for b of -1 corresponds to perfect convergence:
     - Higher initial income on average lowers subsequent growth one-for-one.
   - A value of b of 0 implies that growth is uncorrelated with initial income and thus there is no convergence.

Convergence

The results are

$$ln[(\frac{Y}{N})_{i,1979}] - ln[(\frac{Y}{N})_{i,1870}] = 8.457 - \underset{(0.094)}{0.995} ln[(\frac{Y}{N})_{i,1870}] \tag{1.37}$$

$$R^2 = 0.87$$

Convergence

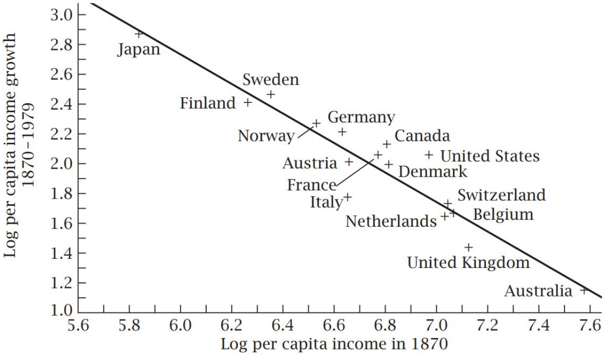


FIGURE 1.7 Initial income and subsequent growth in Baumol's sample (from DeLong, 1988; used with permission)

Convergence

- The regression suggests almost perfect convergence.
  - The estimate of b is almost equal to -1, and it is estimated precisely.
- DeLong(1988) demonstrates, however, that Baumol's finding is largely spurious. There are two problems:
  1. *Sample Selection*.
     - Since historical data are constructed retrospectively, the countries that have long data series are generally those that are the most industrialized today.
     - Countries that were not rich 100 years ago are typically in the sample only if they grew rapidly over the next 100 years.
     - Countries that were rich 100 years ago, in contrast, are generally included even if their subsequent growth was only moderate.

     $\Rightarrow$ We are likely to see poorer countries growing faster than richer ones in the sample of countries we consider, even if there is no tendency for this to occur on average.

Convergence

The natural way to eliminate this selection bias is to use a rule for choosing the sample that is not based on the variable we are trying to explain, which is growth over the period 1870-1979.

- DeLong considers the richest countries as of 1870.
  - This causes him to add seven countries to Baumol's list (Argentina, Chile, East Germany, Ireland, New Zealand, Portugal, and Spain) and to drop one (Japan).

Convergence

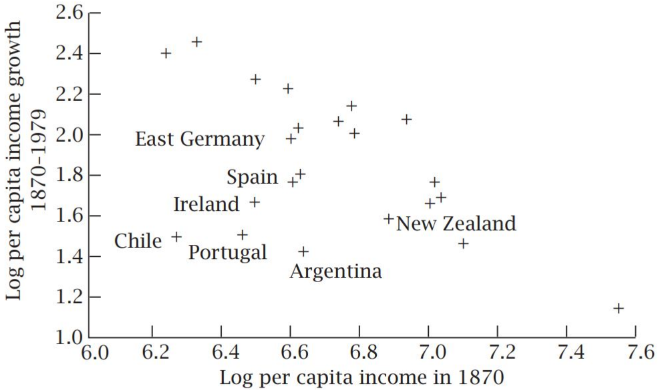


FIGURE 1.8 Initial income and subsequent growth in the expanded sample (from DeLong, 1988; used with permission)

Convergence

- The inclusion of the new countries weakens the case for convergence considerably.
  - The regression now produces an estimate of b of -0.566, with a standard error of 0.144.
  - Accounting for the selection bias in Baumol's procedure eliminates about half of the convergence that he finds.

2. *Measurement error*.
   - Estimates of real income per capita in 1870 are imprecise.
   - Measurement error again creates bias toward fining convergence.
     - When 1870 income is overstated, growth over the period 1870-1979 is understated by an equal amount;
     - When 1870 income is understated, the reverse occurs.

     $\Rightarrow$ Measured growth tends to be lower in countries with higher measured initial income even if there is no relation between actual growth and actual initial income.

Convergence

DeLong therefore considers the following model:

$$ln[(\frac{Y}{N})_{i,1979}] - ln[(\frac{Y}{N})_{i,1870}]^* = a + b\ ln[(\frac{Y}{N})_{i,1870}] * + \epsilon_i \tag{1.38}$$

$$ln[(\frac{Y}{N})_{i,1870}] = ln[(\frac{Y}{N})_{i,1870}]^* + u_i \tag{1.39}$$

$ln[(Y/N)_{1870}]^*$ is the true value of log income per capita in 1870;
$ln[(Y/N)_{1870}]$ is the measured value.
ϵ and u are assumed to be uncorrelated with each other and with $ln[(Y/N)_{1870}]^*$.

Convergence

Even moderate measurement error has a substantial impact on the results.

- For the unbiased sample, the estimate of b reaches 0 (no tendency toward convergence) for $\sigma_u \simeq 0.15$, and is 1 (tremendous divergence) for $\sigma_u \simeq 0.20$.

  $\Rightarrow$ Plausible amounts of measurement error eliminate most or all of the remainder of Baumol's estimate of convergence.

It is also possible to investigate convergence for different samples of countries and different time periods.

Convergence

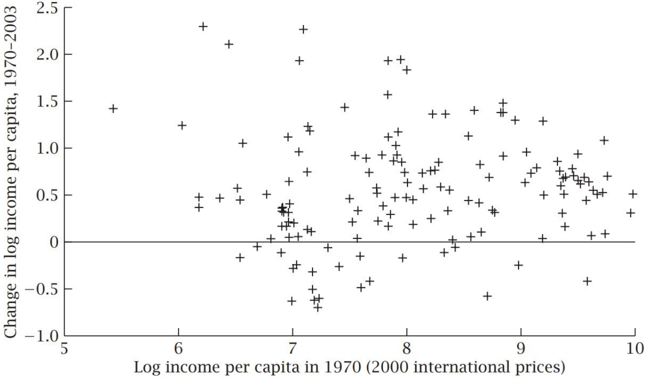


FIGURE 1.9 Initial income and subsequent growth in a large sample

References

- Romer, D. (2012). Chapter 1. Advanced macroeconomics (4th ed). McGraw-Hill/Irwin.