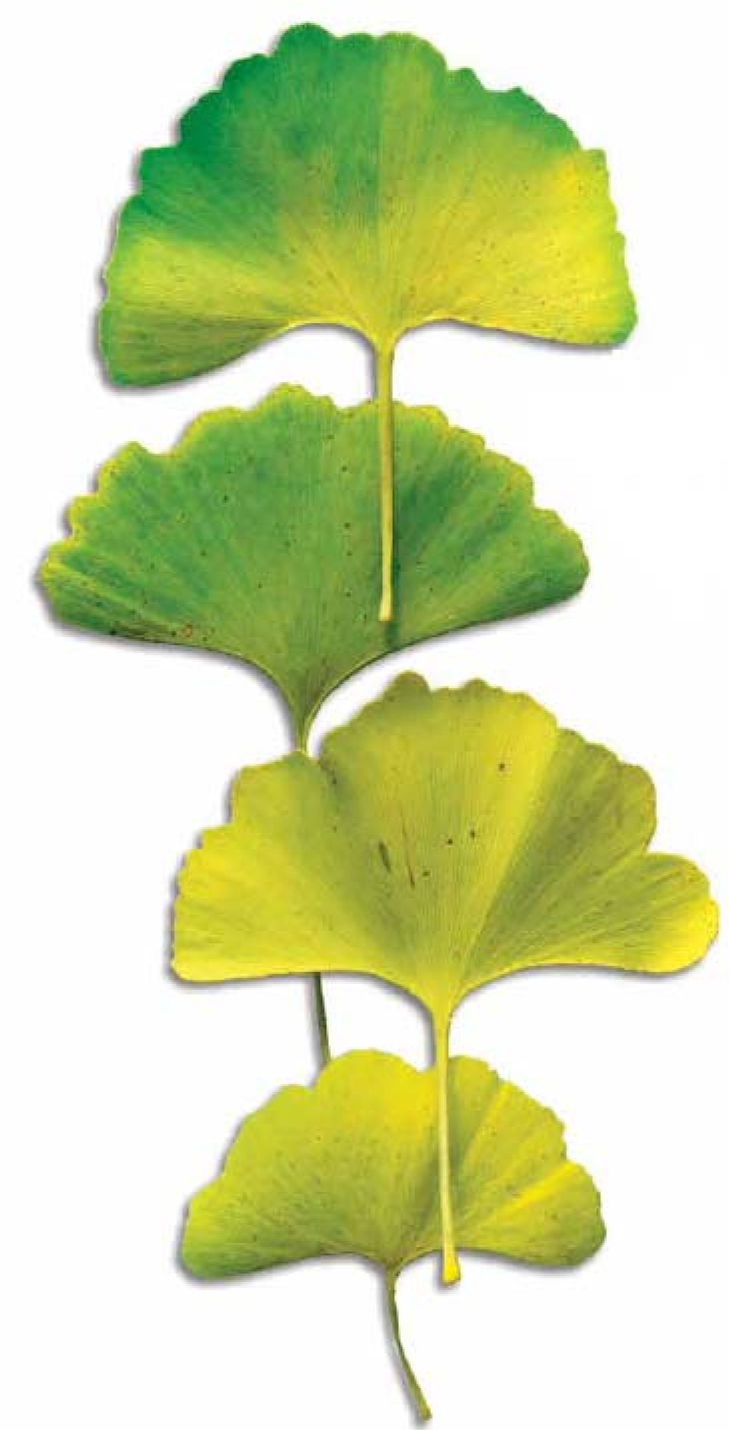


Chapter 27

Oligopoly Models

Chapter Outline

- Some Specific Oligopoly Models : Cournot, Bertrand and Stackelberg
- Competition When There are Increasing Returns to Scale
- Monopolistic Competition
- A Spatial Interpretation of Monopolistic Competition
- Historical Note: Hotelling's Hot Dog Vendors
- Consumer Preferences and Advertising

Models of Oligopoly

- An **oligopoly** is a market with only a few important sellers.
  - A **duopoly** is an oligopoly with only two firms.
- Compared to perfect competition
  - Firms face downward sloping demand and thus can choose their price.
  - The market contains sufficiently few firms that each firm recognizes that the price it will receive for its output depends on how much output it chooses to produce.

Models of Oligopoly

- Compared to monopoly
  - An oligopoly has more than one firm. Hence, in an oligopoly, the optimal decision of one firm depends on the decision made by other firms in the market.
  - Such strategic interaction between firms does not occur in a monopoly since no other firms exist in the market (except interaction with a potential entrant).

Cournot Model

- ***Cournot model:*** oligopoly model in which each firm assumes that rivals will continue producing their current output levels.
  - Firms pick quantities simultaneously.
  - Each firm treats the other's quantity as a fixed number, one that will not respond to its own production decisions.

Cournot Model

- Each firm chooses its quantity of output to maximize its profits, taking the other firm's output as given.
- Given the rival's quantity, the firm maximizes its profits by picking the quantity where marginal revenue equals marginal cost $MR = MC$ (like a monopolist facing the residual demand curve).
- Each firm's quantity depends on the quantity of its rival, a relationship known as a reaction curve.
- The equilibrium quantities for the two firms occur at intersection of the two reaction curves, one for each firm.

Residual Demand Curve

- Figure 13.1 shows how to determine a Cournot duopolist's optimal output, for a given output of the other firm, based on the residual demand curve.
- The residual demand curve is the portion of the market demand curve that remains for the first firm after the second firm has already sold its output.

Cournot Duopoly

- The residual demand curve facing the first firm is found by shifting the vertical axis over to the right by the amount of the second firm's output Q_2.
- For the linear demand curve $P = a - bQ$, the residual demand curve facing the first firm would be

$$
\begin{aligned}
P &= a - bQ \\
&= a - b(Q_1 + Q_2) \\
&= a - bQ_2 - bQ_1
\end{aligned}
$$

where $a - bQ_2$ is the price intercept.

Cournot Duopoly

- The marginal revenue curve for the first firm is found as if the first firm were a monopolist facing the residual demand curve $P = a - bQ_2 - bQ_1$
  - marginal revenue has the same intercept but twice the slope. curve $MR = a - bQ_2 - 2bQ_1$
- Remember to double only the coefficient on the first firm's output (not the output of the other firm).
- Graphically, the marginal revenue curve departs from the demand curve at the point $Q = Q_2$ and $P = a - bQ_2$, where the first firm produces no output.

Cournot Duopoly

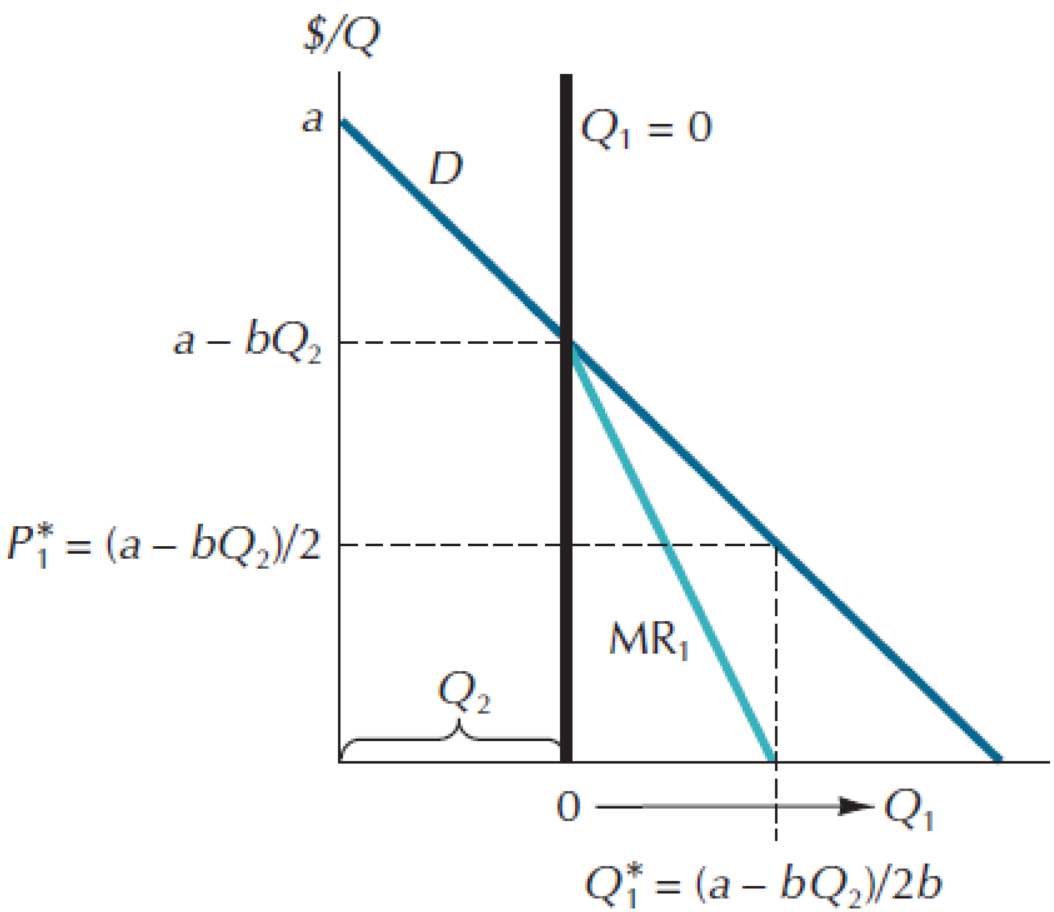
- Figure 13.1 covers the special case of linear demand with zero marginal cost.
- To maximize profits, set $MC = MR$ and solve for Q_1:

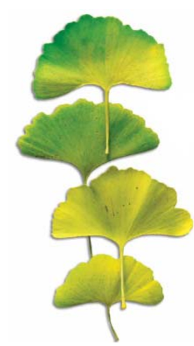
$$0 = a - bQ_2 - 2bQ_1$$

$$2bQ_1 = a - bQ_2$$

$$Q_1 = \frac{a}{2b} - \frac{1}{2}Q_2$$

Figure 13.1: Profit-Maximizing Cournot Duopolist





Cournot Model

- ***Reaction function:*** a curve that tells the profit-maximizing level of output for one oligopolist for each amount supplied by another.

Cournot Duopoly

- For Cournot duopoly with linear demand and zero marginal costs, the reaction function for firm one is

$$R_1(Q_2) = Q_1 = \frac{a}{2b} - \frac{1}{2}Q_2$$

- The reaction function for firm two is correspondingly

$$R_2(Q_1) = Q_2 = \frac{a}{2b} - \frac{1}{2}Q_1$$

Cournot Duopoly

- On a graph of output of firm one versus output of firm two, the reaction curve for firm one $R_1(Q_2) = Q_1 = \frac{a}{2b} - \frac{1}{2} Q_2$ has
  - vertical intercept $Q_1 = \frac{a}{2b}$ when $Q_2 = 0$ and
  - horizontal intercept $Q_2 = \frac{a}{b}$ when $Q_1 = 0$.
- And similarly for the reaction function for firm two.

Cournot Duopoly

- To find the equilibrium output of firm one and of firm two where the two reaction functions intersect, solve $Q_2 = \frac{a}{2b} - \frac{1}{2}Q_1$ for $Q_1 = \frac{a}{b} - 2Q_2$ then set equal to

$$Q_1 = \frac{a}{2b} - \frac{1}{2}Q_2$$

$$\frac{a}{b} - 2Q_2 = \frac{a}{2b} - \frac{1}{2}Q_2$$

$$\frac{3}{2}Q_2 = \frac{a}{2b}$$

$$\mathrm{Q}_1 = Q_2 = \frac{a}{3b}$$

Cournot Duopoly

- Or alternatively insert $Q_2 = \frac{a}{2b} - \frac{1}{2}Q_1$ into

$$Q_1 = \frac{a}{2b} - \frac{1}{2}Q_2$$

$$Q_1 = \frac{a}{2b} - \frac{1}{2}\left(\frac{a}{2b} - \frac{1}{2}Q_1\right)$$

$$\frac{3}{4}Q_1 = \frac{a}{4b}$$

$$\mathrm{Q}_1 = Q_2 = \frac{a}{3b}$$

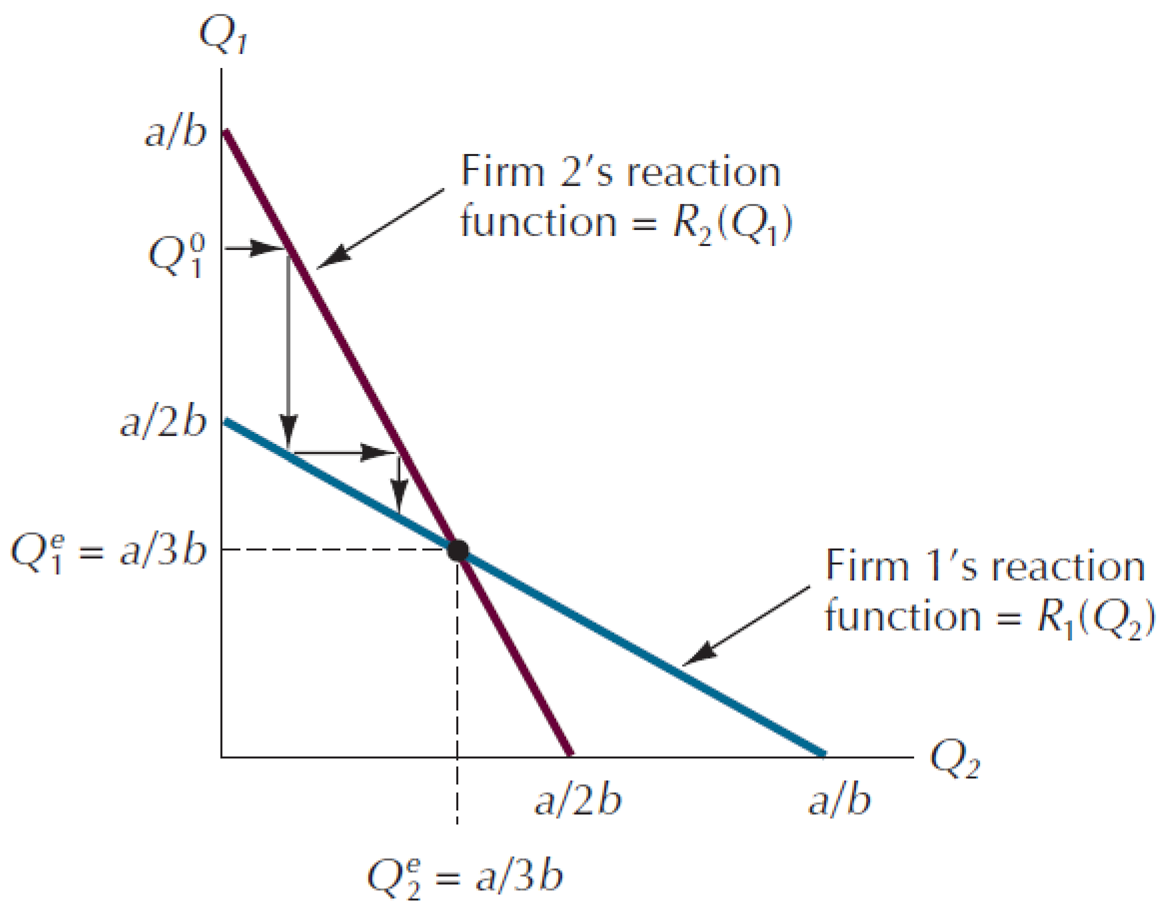
Cournot Duopoly

- With quantity of each firm $Q_1 = Q_2 = \frac{a}{3b}$,
- Total quantity $Q = Q_1 + Q_2 = \frac{2a}{3b}$
- Price $P = a - bQ = a - b\left(\frac{2a}{3b}\right) = \frac{a}{3}$
- Profit $TR - TC = PQ - 0 = \frac{a}{3b}\left(\frac{a}{3}\right) = \frac{a^2}{9b}$
- Industry Profit $\frac{2a^2}{9b}$

Monopoly

- A monopoly facing the same linear demand with zero marginal costs would produce quantity $Q_m = \frac{a}{2b}$, charge price $P_m = \frac{a}{2}$, and profit $\frac{a^2}{4b}$.
- Compared to a **shared monopoly** where each firm would produce half of Q_m, a Cournot duopoly produces more, charges a lower price, and earns less profit.

Figure 13.2: Reaction Functions for Cournot Duopolists



Cournot Duopoly P = 56 – 2Q

- Suppose have linear demand curve $P = 56 - 2Q$ with constant marginal cost $MC = 20$.
- Firm one's marginal revenue

$$MR = 56 - 2Q_2 - 4Q_1$$

- Set marginal revenue equal to marginal cost and solve for firm one's output in terms of firm two's output (firm one's reaction curve)

$$20 = 56 - 2Q_2 - 4Q_1$$

$$R_1(Q_2) = Q_1 = 9 - \frac{1}{2}Q_2$$

Cournot Duopoly P = 56 – 2Q

- By symmetry, firm two's reaction curve is

$$R_2(Q_1) = Q_2 = 9 - \frac{1}{2}Q_1$$

- Substitute firm two's reaction curve into firm one's to find firm one's optimal quantity

$$Q_1 = 9 - \frac{1}{2}Q_2$$

$$Q_1 = 9 - \frac{1}{2}\left(9 - \frac{1}{2}Q_1\right)$$

$$Q_1 = Q_2 = 6$$

Cournot Duopoly P = 56 – 2Q

- Quantity of each firm $Q_1 = Q_2 = 6$,
- Total quantity $Q = Q_1 + Q_2 = 12$
- Price $P = 56 - 2Q = 56 - 2(12) = 32$
- Profit $(P - c)Q = (32 - 20)6 = 72$
- Industry Profit $2(72) = 144$

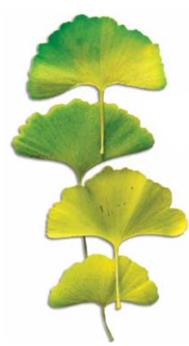
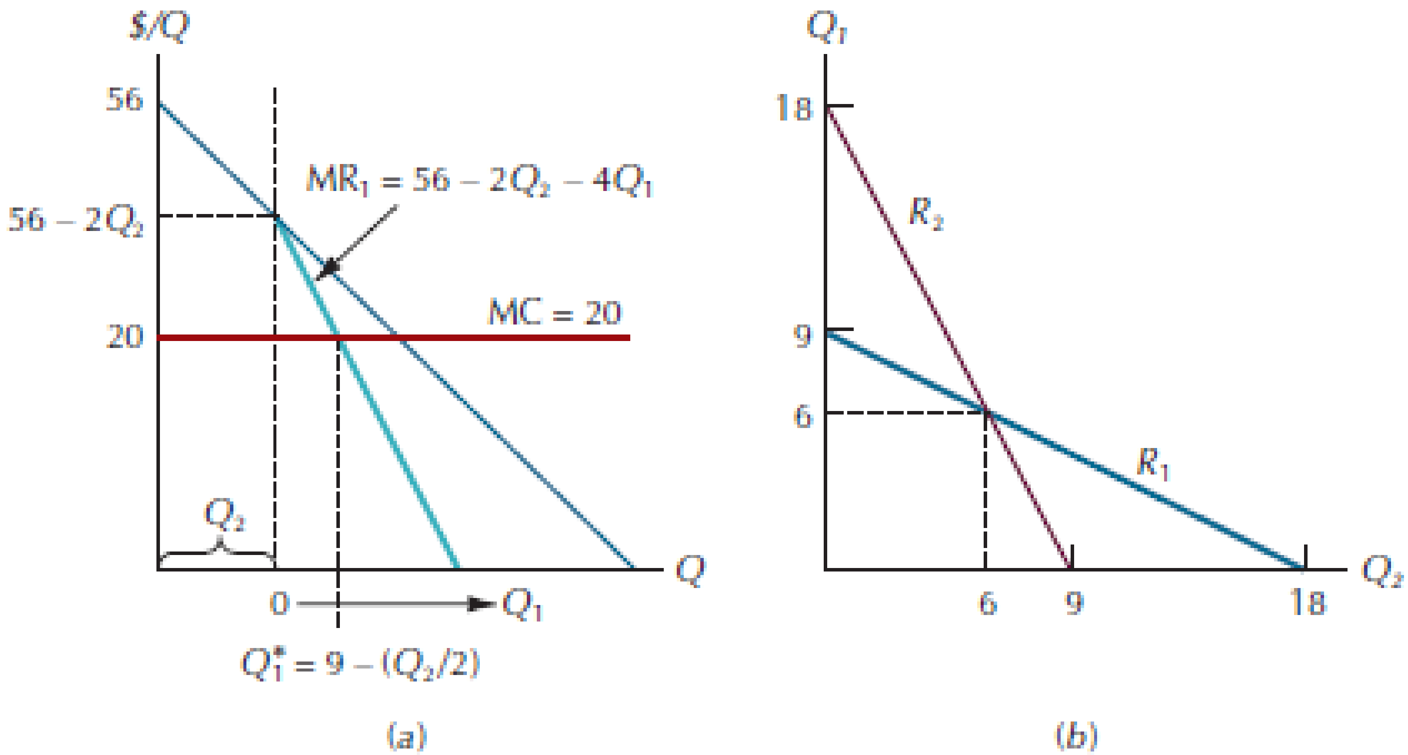


Figure 13.3: Deriving the Reaction Functions for Specific Duopolists



Bertrand Model

- ***Bertrand model:*** oligopoly model in which each firm chooses its price simultaneously, assuming that rivals will continue charging their current prices.

Bertrand Model

- Strategy: If charging more than other firms, sales zero and need to lower price.
  - If charging the same as other firms, split the market so better to charge a tiny amount less and get the whole market.
- Results in all firms charging price equal to cost and earning no profits.

Bertrand Duopoly P = 56 – 2Q

- Price $P = MC = 20$
- Total quantity $20 = 56 - 2Q, Q = 18$
- Quantity of each firm $Q_1 = Q_2 = 9$
- Profit $(P - c)Q = (20 - 20)9$ =0
- Industry Profit $2(0) = 0$

Stackelberg Model

- ***Stackelberg model:*** oligopoly model in which one firm (the leader) picks its quantity before the other firm (the follower).

Stackelberg Duopoly

- The leader firm 1 knows that follower firm two will pick output according to the Cournot reaction function.
  - For linear demand with zero marginal costs

$$Q_2 = \frac{a}{2b} - \frac{1}{2}Q_1$$

Stackelberg Duopoly

- Residual demand facing the leader is

$$P = a - bQ_2 - bQ_1$$

$$P = a - b\left(\frac{a}{2b} - \frac{1}{2}Q_1\right) - bQ_1$$

$$P = \frac{a}{2} - \frac{b}{2}Q_1$$

- Marginal revenue

$$MR = \frac{a}{2} - bQ_1$$

Stackelberg Duopoly

- Pick quantity to set marginal revenue for leader equal to zero marginal cost

$$0 = \frac{a}{2} - bQ_1$$

$$Q_1 = \frac{a}{2b}$$

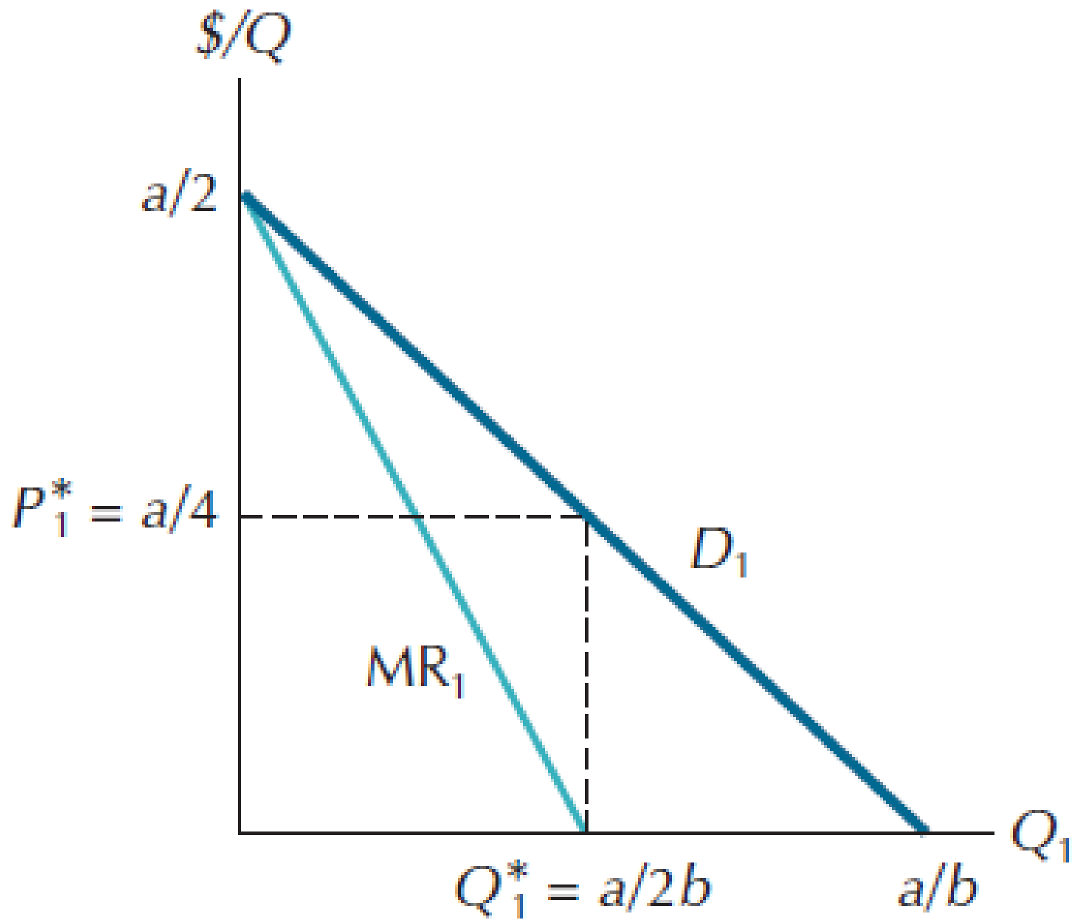
- The follower firm 2 will respond by picking

$$Q_2 = \frac{a}{2b} - \frac{1}{2}Q_1 = \frac{a}{2b} - \frac{1}{2}\left(\frac{a}{2b}\right) = \frac{a}{4b}$$

Stackelberg Duopoly

- Total quantity $Q = Q_1 + Q_2 = \frac{a}{2b} + \frac{a}{4b} = \frac{3a}{4b}$
- Price $P = a - bQ = a - b\left(\frac{3a}{4b}\right) = \frac{a}{4}$
- Leader Profit $PQ_1 = \frac{a}{4}\left(\frac{a}{2b}\right) = \frac{a^2}{8b}$
- Follower Profit $PQ_2 = \frac{a}{4}\left(\frac{a}{4b}\right) = \frac{a^2}{16b}$
- Industry Profit $\frac{3a^2}{16b}$

Figure 13.4: The Stackelberg Leader's Demand and Marginal Revenue Curves



Stackelberg Duopoly P = 56 – 2Q

- Residual demand facing the leader is

$$P = 56 - 2Q_2 - 2Q_1$$

$$P = 56 - 2\left(9 - \frac{1}{2}Q_1\right) - 2Q_1$$

$$P = 38 - Q_1$$

- Marginal revenue

$$MR = 38 - 2Q_1$$

Stackelberg Duopoly P = 56 – 2Q

- Pick quantity to set marginal revenue for leader equal to marginal cost

$$20 = 38 - 2Q_1$$

$$Q_1 = 9$$

- The follower firm 2 will respond by picking

$$Q_2 = 9 - \frac{1}{2}(9) = 4.5$$

Stackelberg Duopoly P = 56 - 2Q

- Total quantity $Q = Q_1 + Q_2 = 9 + 4.5 = 13.5$
- Price $P = 56 - 2Q = 56 - 2(13.5) = 29$
- Leader Profit $(P - c)Q_1 = (29 - 20)9 = 81$
- Follower Profit $(P - c)Q_2 = (29 - 20)4.5 = 40.5$
- Industry Profit $81 + 40.5 = 121.5$

Figure 13.5: The Stackelberg Equilibrium

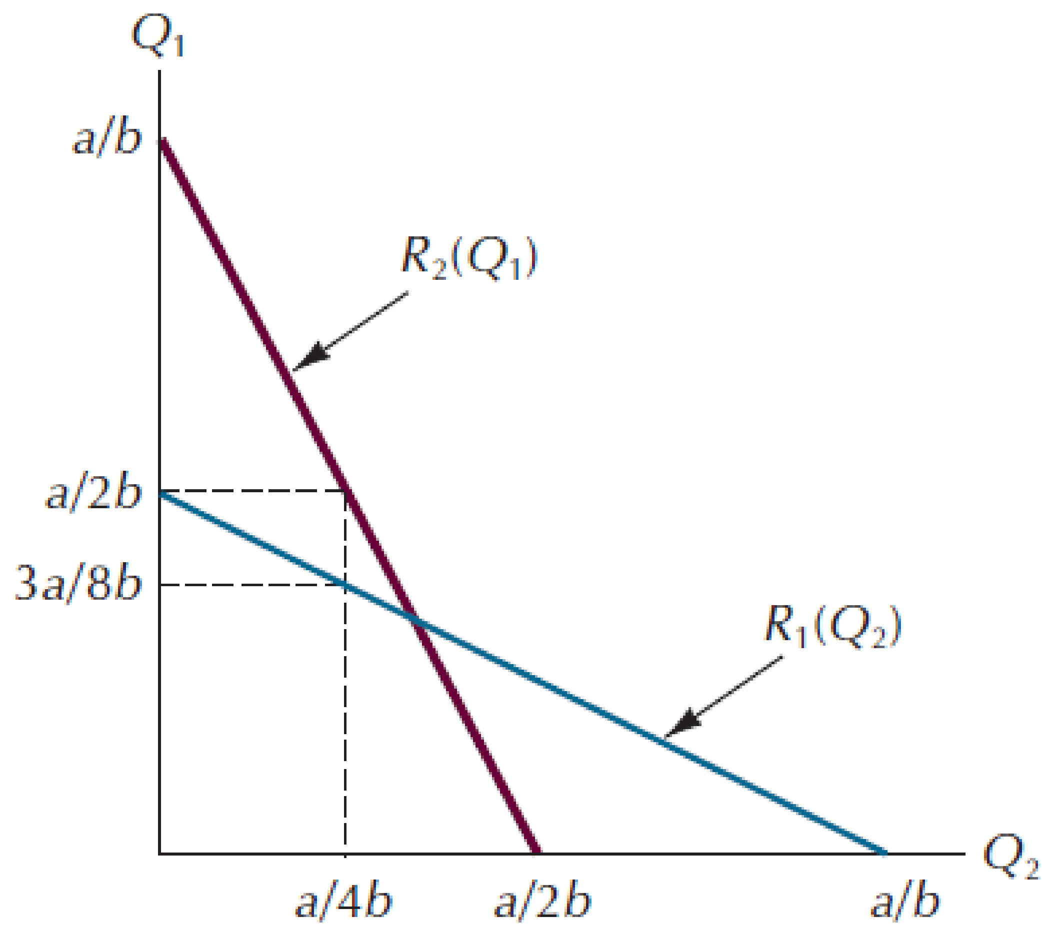


Table 13.1: Comparison Of Outcomes

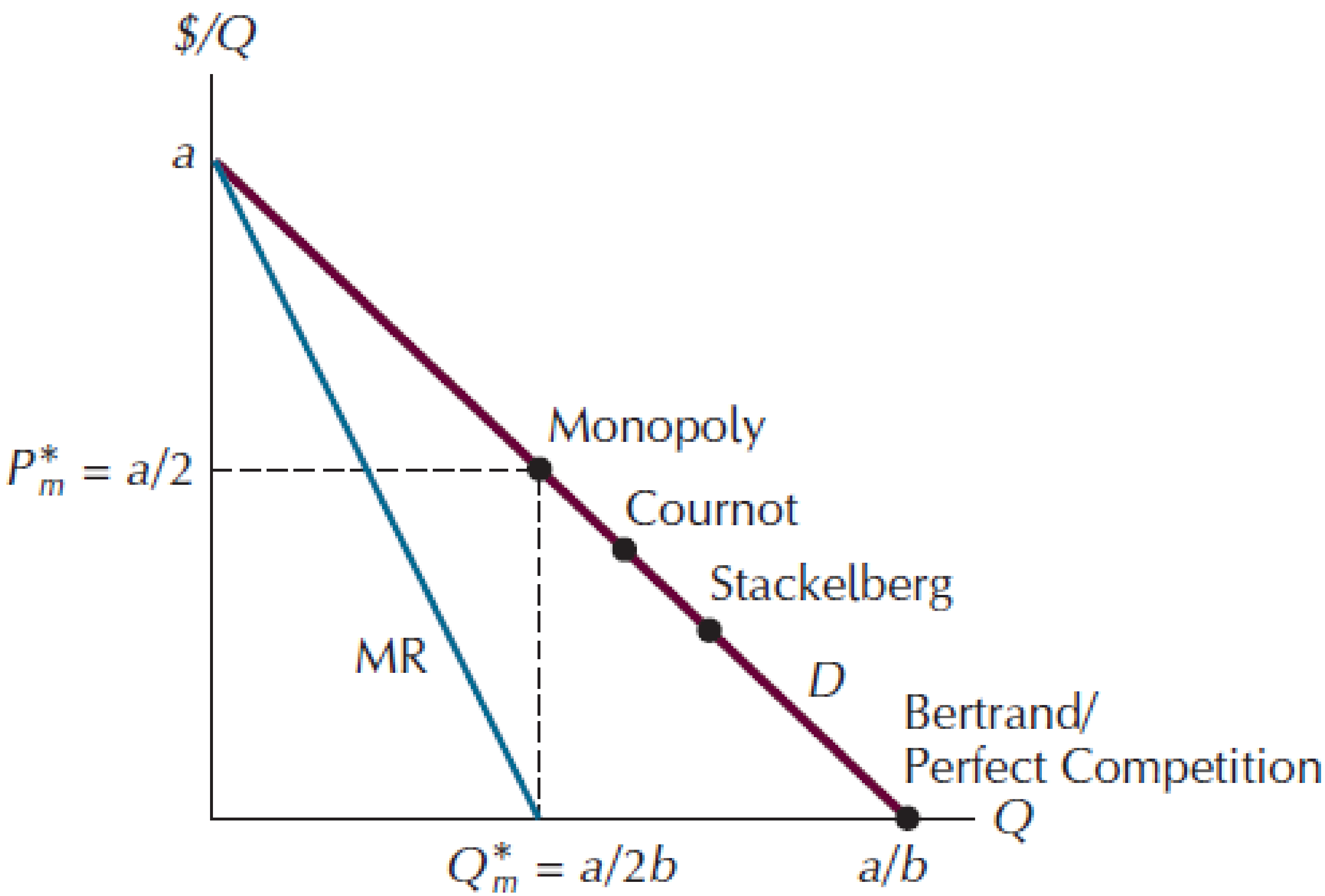
TABLE 13.1

Comparison of Oligopoly Models

Model	Industry output Q	Market price *P*	Industry profit Π
Shared monopoly	$Q_m = a/(2b)$	$P_m = a/(2)$	$\Pi_m = a^2/(4b)$
Cournot	$(4/3)Q_m$	$(2/3)P_m$	$(8/9)\Pi_m$
Stackelberg	$(3/2)Q_m$	$(1/2)P_m$	$(3/4)\Pi_m$
Bertrand	$2Q_m$	0	0
Perfect competition	$2Q_m$	0	0

All four models assume a market demand curve of $P = a - bQ$ and marginal cost equal to zero. (Of course, if marginal cost is not zero, the entries will be all different from the ones shown.)

Figure 13.6: Comparing Equilibrium Price and Quantity



Comparison for P = 56 – 2Q

MC = 20	Quantity Q	Price P	Industry Profit
Monopoly	8	40	160
Cournot	12	32	144
Stackelberg	13.5	29	121.5
Bertrand, Perfect Competition	18	20	0

Problem 1

1. The market demand curve for a pair of Cournot duopolists is given as $P = 36 - 3Q$, where $Q = Q_1 + Q_2$. The constant per unit marginal cost is 18 for each duopolist. Find the Cournot equilibrium price, quantity, and profits.

Solution 1

1. Linear demand curve $P = 36 - 3Q$ with constant marginal cost $MC = 18$. Firm one's marginal revenue

$$MR = 36 - 3Q_2 - 6Q_1$$

Set marginal revenue equal to marginal cost and solve for firm one's output in terms of firm two's output (firm one's reaction curve)

$$18 = 36 - 3Q_2 - 6Q_1$$

$$R_1(Q_2) = Q_1 = 3 - \frac{1}{2}Q_2$$

Solution 1

By symmetry, firm two's reaction curve is

$$R_2(Q_1) = Q_2 = 3 - \frac{1}{2}Q_1$$

Substitute firm two's reaction curve into firm one's to find firm one's optimal quantity

$$Q_1 = 3 - \frac{1}{2}Q_2$$

$$Q_1 = 3 - \frac{1}{2}\left(3 - \frac{1}{2}Q_1\right)$$

$$Q_1 = Q_2 = 2$$

Solution 1

Quantity of each firm $Q_1 = Q_2 = 2$

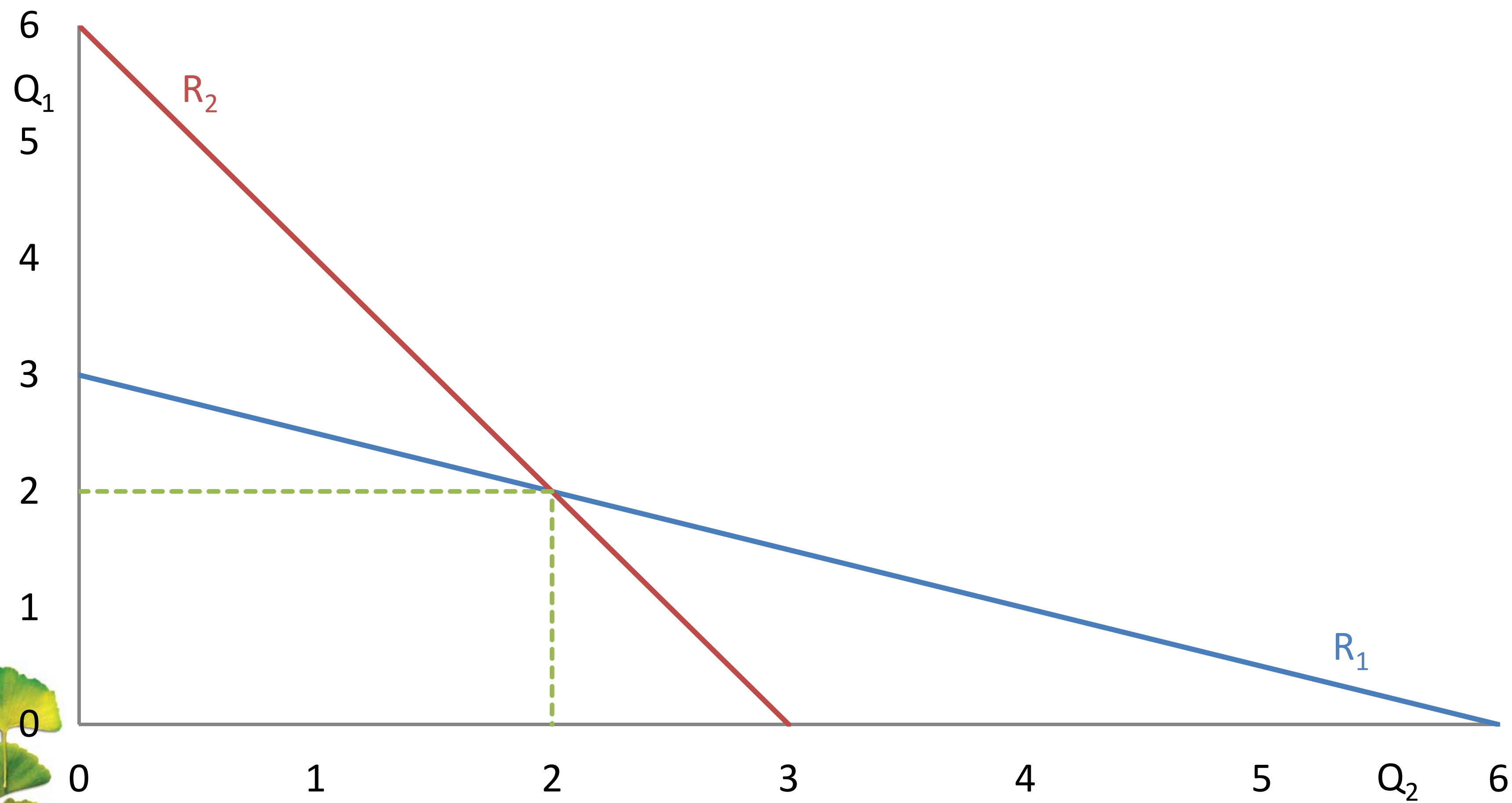
Total quantity $Q = Q_1 + Q_2 = 4$

Price $P = 36 - 3Q = 36 - 3(4) = 24$

Profit $(P - c)Q = (24 - 18)2 = 12$

Industry Profit $2(12) = 24$

Solution 1 Figure



Problem 2

2. The Zambino brothers enjoy a monopoly of the U.S. market for public fireworks displays for crowds above a quarter of a million. The annual demand for these fireworks displays is $P = 140 - Q$. The marginal cost is $20. A family dispute broke the firm in two. Alfredo Zambino now runs one firm and Luigi Zambino runs the other. They still have the same marginal costs, but now they are Cournot duopolists. How much profit has the family lost?

Solution 2

2. Linear demand curve $P = 140 - Q$ with constant marginal cost $MC = 20$. Monopoly's marginal revenue

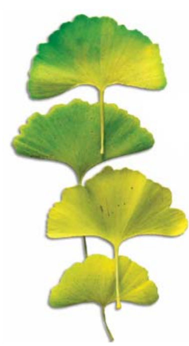
$$MR = 140 - 2Q$$

Set marginal revenue equal to marginal cost and solve for output

$$20 = 140 - 2Q, Q_m = 60$$

$$P_m = 140 - Q = 140 = 60 = 80$$

$$(P - c)Q = (80 - 20)60 = 3600$$



Solution 2

As separate Cournot duopolists, firm one's marginal revenue

$$MR = 140 - Q_2 - 2Q_1$$

Set marginal revenue equal to marginal cost and solve for firm one's output in terms of firm two's output (firm one's reaction curve)

$$20 = 140 - Q_2 - 2Q_1$$

$$R_1(Q_2) = Q_1 = 60 - \frac{1}{2}Q_2$$

Solution 2

By symmetry, firm two’s reaction curve is

$$R_2(Q_1) = Q_2 = 60 - \frac{1}{2}Q_1$$

Substitute firm two’s reaction curve into firm one's to find firm one’s optimal quantity

$$Q_1 = 60 - \frac{1}{2}Q_2$$

$$Q_1 = 60 - \frac{1}{2}\left(60 - \frac{1}{2}Q_1\right)$$

$$Q_1 = Q_2 = 40$$

Solution 2

Quantity of each firm $Q_1 = Q_2 = 40$

Total quantity $Q = Q_1 + Q_2 = 80$

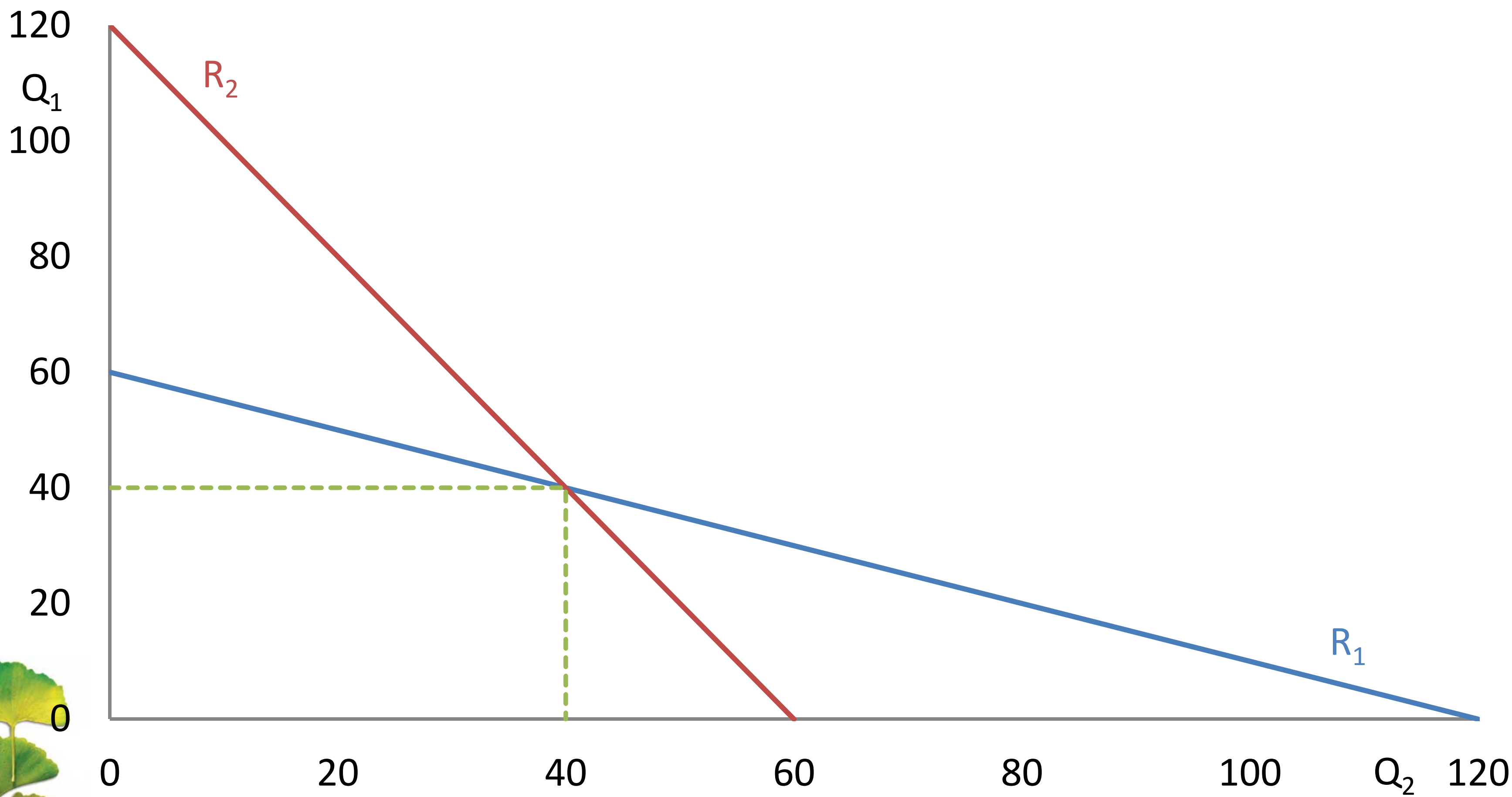
Price $P = 140 - Q = 140 - 80 = 60$

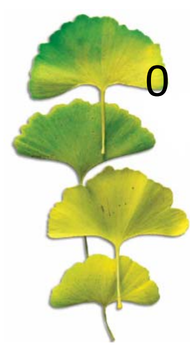
Profit $(P - c)Q = (60 - 20)40 = 1600$

Industry Profit $2(1600) = 3200$

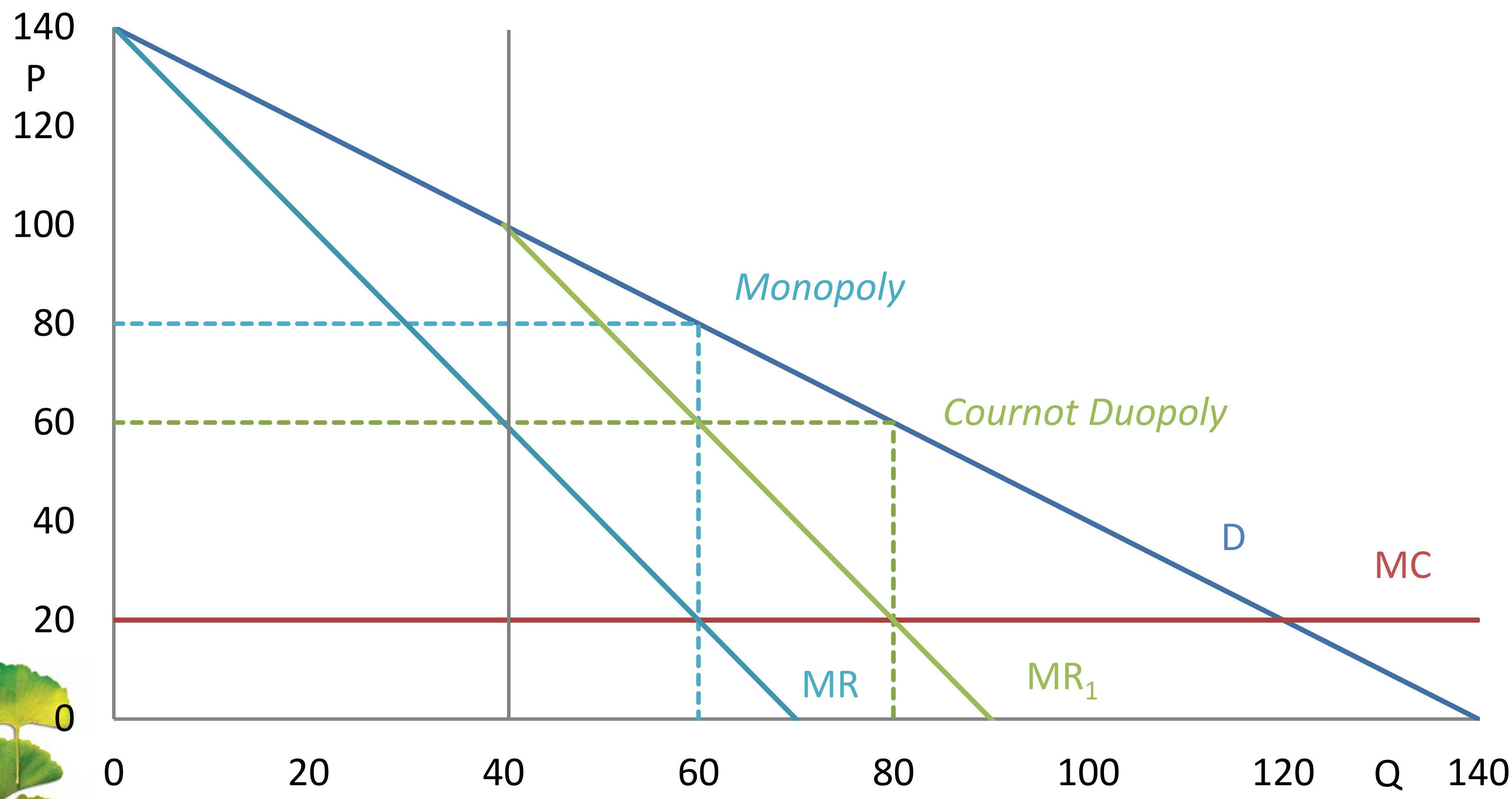
Profit loss $3600 - 3200 = 400$

Solution 2 Figure





Solution 2 Figure



Problem 3

3. The market demand for mineral water is given by $P = 15 - Q$. There are two firms that produce mineral water, each with a constant marginal cost of 3 per unit. Find the Cournot equilibrium price, quantity, and profits.

Solution 3

3. Linear demand curve $P = 15 - Q$ with constant marginal cost $MC = 3$. Firm one's marginal revenue

$$MR = 15 - Q_2 - 2Q_1$$

Set marginal revenue equal to marginal cost and solve for firm one's output in terms of firm two's output (firm one's reaction curve)

$$3 = 15 - Q_2 - 2Q_1$$

$$R_1(Q_2) = Q_1 = 6 - \frac{1}{2}Q_2$$

Solution 3

By symmetry, firm two's reaction curve is

$$R_2(Q_1) = Q_2 = 6 - \frac{1}{2}Q_1$$

Substitute firm two's reaction curve into firm one's to find firm one's optimal quantity

$$Q_1 = 6 - \frac{1}{2}Q_2$$

$$Q_1 = 6 - \frac{1}{2}\left(6 - \frac{1}{2}Q_1\right)$$

$$Q_1 = Q_2 = 4$$

Solution 3

Quantity of each firm $Q_1 = Q_2 = 4$

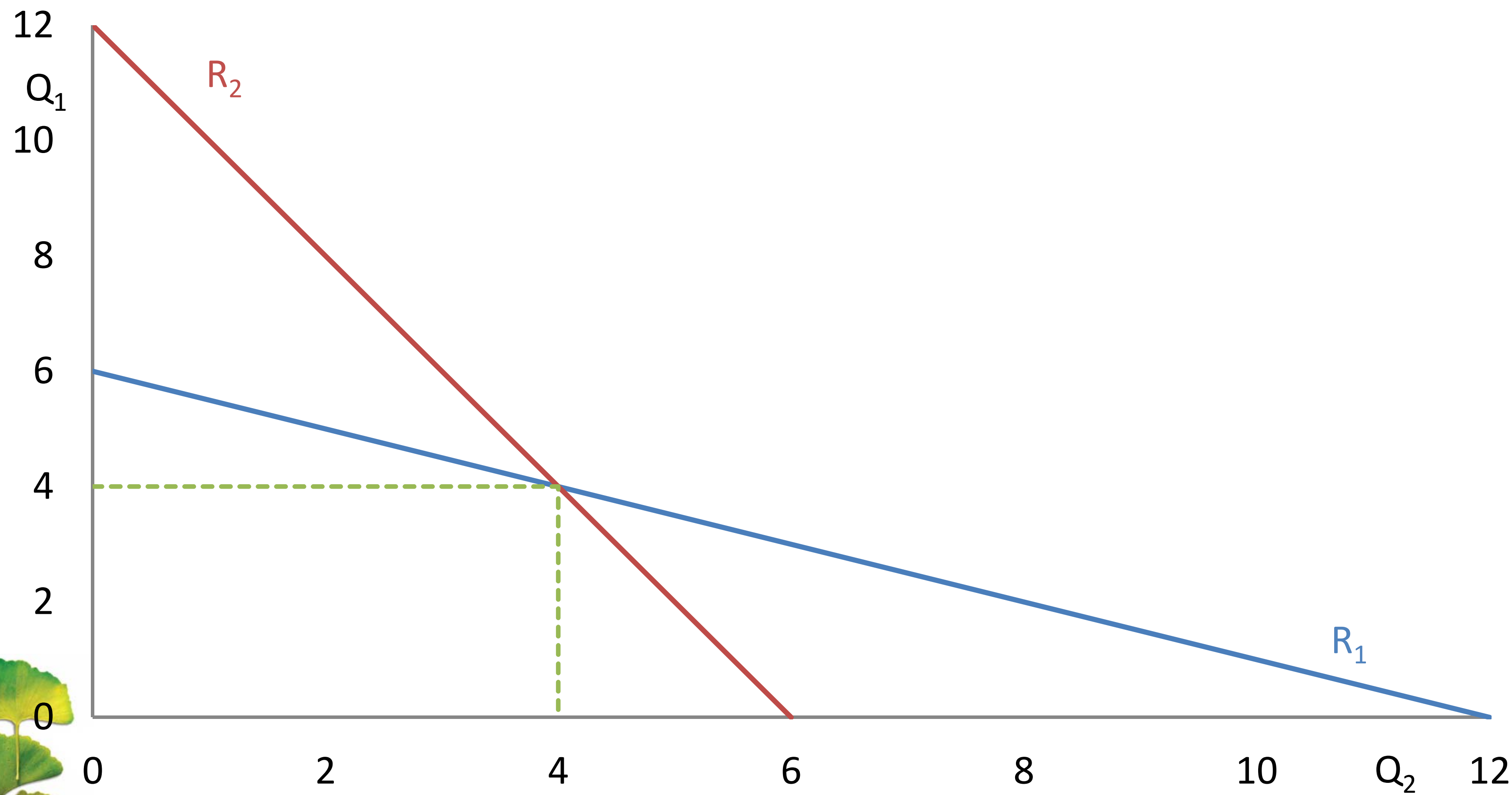
Total quantity $Q = Q_1 + Q_2 = 8$

Price $P = 15 - Q = 15 - 8 = 7$

Profit $(P - c)Q = (7 - 3)4 = 16$

Industry Profit $2(16) = 32$

Solution 3 Figure



Problem 4

4. How would the equilibrium price, quantity, and profits differ if instead the two mineral water firms behaved in a Bertrand fashion?

Solution 4

4. Price $P = MC = 3$

   Total quantity $3 = 15 - Q, Q = 12$

   Quantity of each firm $Q_1 = Q_2 = 6$

   Profit $(P - c)Q = (3 - 3)6 = 0$

   Industry Profit $2(0) = 0$