Microeconomics I

Homework 8

Question 1.

The short-term production function of a competitive firm is given by $Q=f(L)= 6L^{2/3}$, where L is the amount of labor it uses. The cost per unit of labor is w=6 and the price per unit of output is p=3.

a. On the L-Q coordinate plane, plot the graph of the production function.
b. Note the profit is π =pQ-wL. Write down the equation of isoprofit lines.
c. On the same graph, draw three isoprofit lines: (i) one that passes through the point (0,12), (ii) one that passes through the point (0,8), and (iii) one that passes through the point (0,4). What is the slope of the isoprofit lines? How many points on the isoprofit line through (0,12) consist of input-output pairs that are actually feasible?
d. How many units of labor will the firm hire, i.e. what is the optimal level of labor? How much output will it produce? Show them on the graph.
e. Now, solve the firm's optimization problem using calculus. Hint: Plug the production function into the profit function and differentiate with respect to L.

Question 2.

A Los Angeles firm uses a single input to produce a recreational commodity according to a production function $f(x)=4x^{1/2}$, where x is the number of units of input. The price of the commodity is $100 per unit, and the input cost is $50 per unit.

a. Write down the firm's profit function.
b. Find the profit maximizing amounts of input and output. What is the maximum profit?
c. Suppose that the firm is taxed at $20 per unit of its output (note it is a quantity tax) and the price of its input is subsidized by $10 per unit. What is the new input and output levels? What is the new maximal profit?

Question 3.

Suppose in the long-rung the production function of a *competitive* firm given in Question 1 becomes by $Q=f(L,K)= L^{2/3}K^{1/4}$, where L is the amount of labor and K is the amount of capital. The cost per unit of labor is w and the cost of capital is r, which is the interest rate. The price per unit of output is p.

a. Write down the profit as a function of K and L.
b. Find the marginal products of K and L: MP_K and MP_L
c. Now multiply MP_K and MP_L by the output price, p, to get the marginal revenues of capital and labor, respectively.
d. The optimal levels of inputs satisfy the condition: marginal revenue of each input equals its marginal cost. The marginal cost of capital is r, and the marginal cost of labor is w. Solve the resulting system of two equations for the profit-maximizing levels of inputs. What is the profit maximizing level of output? What is the maximum profit?
e. Does this production function exhibit increasing, decreasing or constant returns to scale?
f. If instead of above production function, we have $f(L,K)= L^{2/3}K^{2/3}$ for a competitive firm, what will be the profit-maximizing level of the output? What is the returns to scale of this new production function?

g. Finally, solve the profit-maximization problem for a competitive firm if the production function is exhibiting constant returns to scale, and has the form $f(L,K)= L^{2/3}K^{1/3}$. How many solutions does this problem have? What is the maximum profit that the competitive firm can earn?