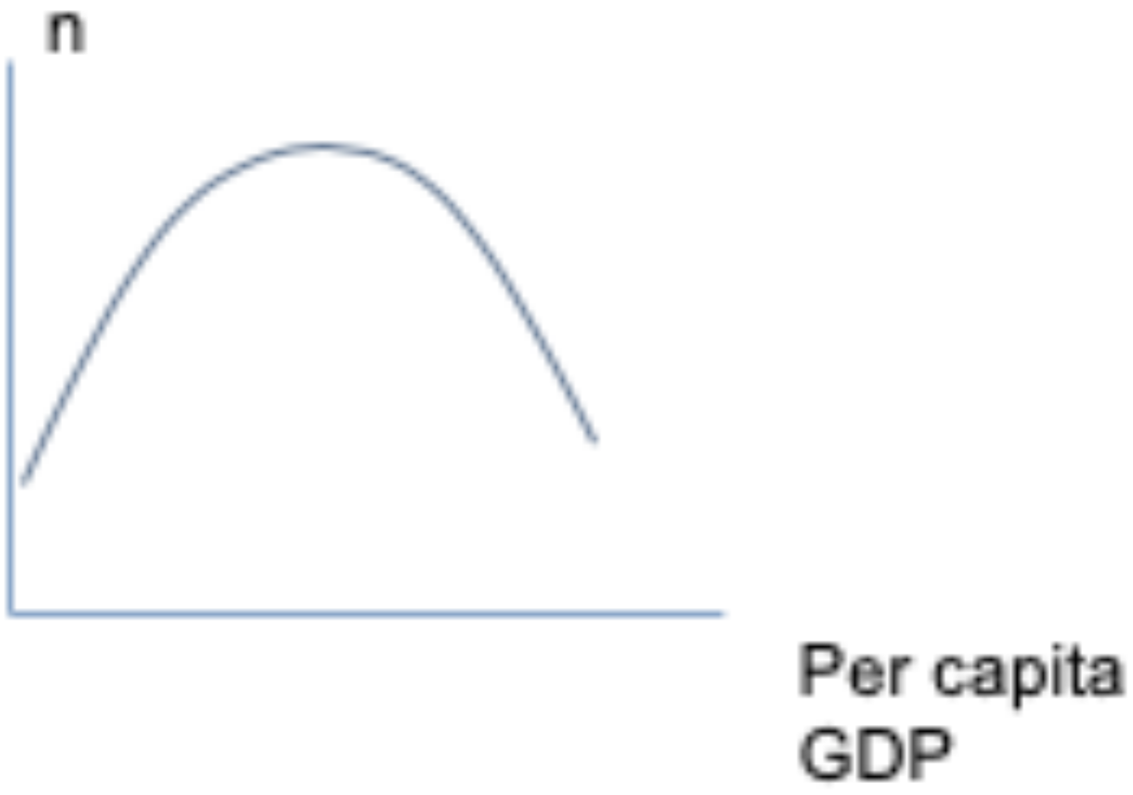
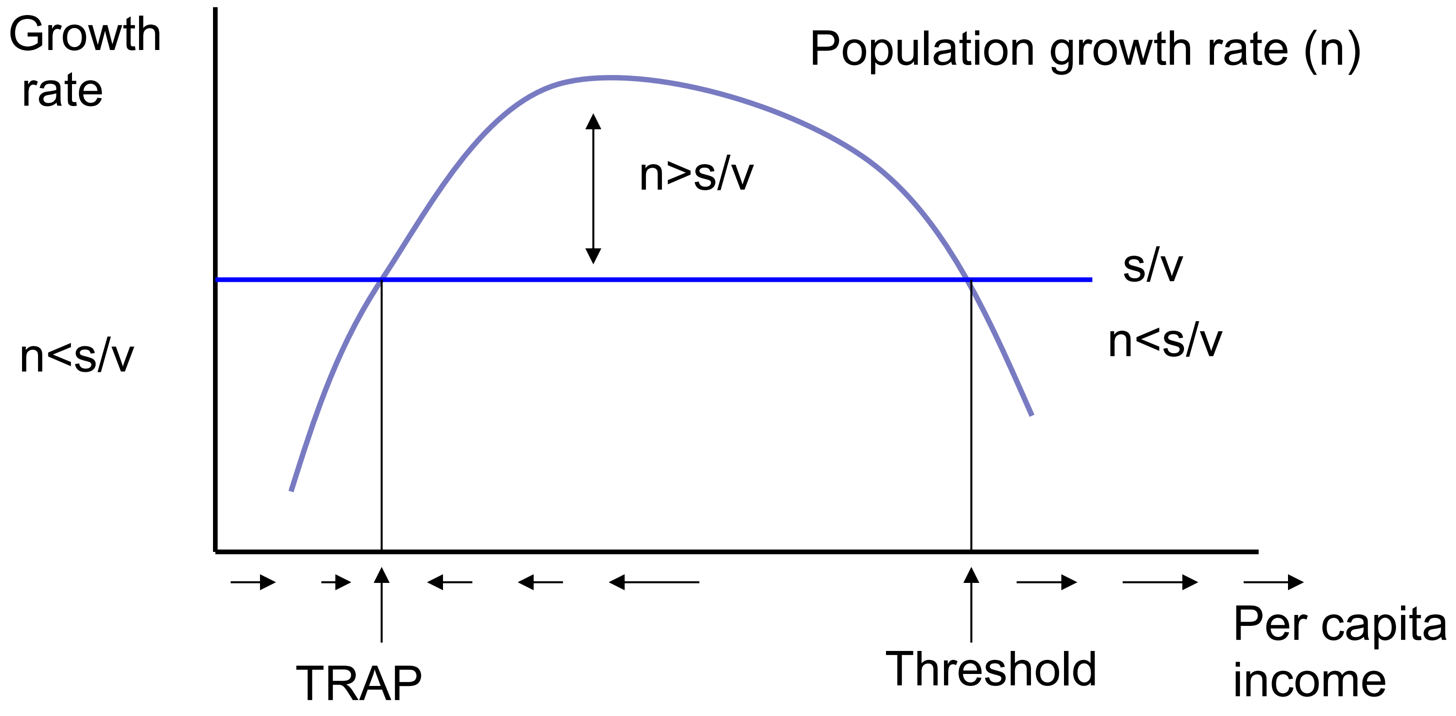
- Modified Harrod-Domar model

Modify the Harrod-Domar model
(Endogenous population growth rate)

- In the Harrod-Domar model assume that population growth rate (*n*) is no longer constant.
- Assume that when an economy becomes richer, *n* first rises and then after a point it starts declining. *n* will vary with *t*.
- Equilibrium condition :
  $\Delta y_t / y_t = s/v - n_t = 0$



Modified H-D model: the trap and threshold model



Either stagnate or grow forever

- Until the threshold level of income is exceeded, the economy will not grow steadily; in fact it will fall back into a low income trap.

- Once the threshold is crossed, the economy will keep expanding forever (the positive knife-edge phenomenon)

- The knife-edge problem is reduced here (but does not disappear).

- The foundation is demography-based, and is somewhat ad hoc.

- Solow Model

- (or rather Solow-Swan model)

Neo-classical growth model: The Solow model (1956)

- Neo-classical: meaning equilibrium-based and relying on market mechanisms. The Solow model is the most well known model of this genre.
- Basic Assumptions:
  - Aggregate macro economy is described by a one-sector economy or one aggregate good which is produced by a production function: $F(K, L)$
  - Constant saving rate (s)

Key assumptions of the Solow model

- The capital-output ratio v is not constant. It is rather dependent on the per capita income Y/L (= y)
  - Recall: In Harrod-Domar this was constant causing a knife-edge problem
- So v will vary with time and is to be denoted as v_t.
- Savings rate is constant: s
- Population to grow at a constant rate: n.
- Constant depreciation rate of capital: δ

Key assumptions of the Solow model

- Decreasing returns to scale (DRS) in the production of per capita GDP. That is, if per capita capital (*K/L* or *k*) rises by 100%, per capita GDP (*y*) will rise by less than 100%.
- Write $Y/L = f(K/L)$ (in per capita terms)
- or $y = f(k)$, $f'(k)>0$, $f''(k) <0$ (<u>DRS in terms of *k*</u>)
- As long as capital (per capita) grows, per capita GDP will also grow, as capital (per capita) is the only input.

Example of a CRS function

Suppose production is given by a CRS Cobb-Douglas production function:

$Y = A\,K^{a}L^{1-a}$, $0<a<1$.

Divide both sides by L, and get

$(Y/L) = A\,K^{a}L^{1-a}/\mathrm{L} = A\,K^{a}L^{-a}, = Y = A\,(K/L)^{a}$,

Or $y = A\,k^{a}$,

Since a <1, this function gives decreasing returns to scale in '*k*.'

Solow model

- The capital-output ratio *v* should not be constant.
- Allow population to grow at a constant rate *n*.
- Allow (for the economy) constant returns to scale in (*K*,*L*).
- Production function:

$$Y = F(K,L)$$

- Write $Y/L = F(K/L, 1)$ (in per capita terms)
- Or

→ $y = f(k)$, $f'(k)>0$, $f''(k) <0$ (DRS in terms of *k*)

Solow model

- **Key equilibrium condition**
- $\boldsymbol{S_t = I_t}$ or in ln per capita terms we can write $\boldsymbol{(S/L)_t = (I/L)_t}$

- Equilibrium every time period
- And time t saving is carried forward to t+1 and converted into investment which leads to greater capital stock in t+1

Now expand each term of the equilibrium conditon: Per capita saving → $(S/L)_t = s(Y/L)_t = \boldsymbol{sy_t}$ (1)

Solow model: Investment

- Now consider per capita investment (I_t/L_t)

Investment by definition is: $I_t = K_{t+1} - K_t(1-\delta)$

Rewrite it in per capita term as:

$$(I/L)_t = (K_{t+1} / L_t) - (K_t/L_t)(1-\delta)$$

Or, $$(I/L)_t = (K_{t+1} / L_t) - k_t(1-\delta)$$

Now pay attention to ***(K$_{t+1}$ / L$_t$)***. This can be rewritten as

$$(K_{t+1}/L)_t = \boldsymbol{(K_{t+1} / L_{t+1})(L_{t+1} / L_t)}$$
$$= \boldsymbol{k_{t+1} (L_{t+1} / L_t)}$$

Working of the Solow model

- Assumption: labour L grows at a constant growth rate n, → then we get $L_{t+1} = L_t(1+n)$, or
  - $\boldsymbol{(L_{t+1} / L_t) = 1+n}$ **(2)**
- Now return to the equilibrium condition $\boldsymbol{(S/L)_t = (I/L)_t}$
- Substitute the per capita savings expression (1) and the labour growth equation (2) in the equilibrium condition:
  - $sy_t = [K_{t+1}/L_{t+1}](1+n) - k_t(1-\delta)$
  - $sf(k_t) = k_{t+1}(1+n) - k_t(1-\delta)$
  - $k_{t+1}-k_t = sf(k_t) - n\,k_{t+1} - \delta\,k_t$
- Solow growth equation:

  $\boldsymbol{\Delta k_t = sf(k_t) - nk_{t+1} - \delta k_t}$

Solow growth equation

- $\Delta k_t = sf(k_t) - nk_{t+1} - \delta k_t$

- Steady state growth: Capital, Labour and GDP must grow at the same rate → $k_{t+1}=k_t$ for all t

- Growth in per capita capital (k) and output (y) will be zero in equilibrium → $\Delta k = 0$

- Steady state: Time subscripts do not matter.

- → $\Delta k = sf(k) - k(n+\delta)$
- → $\Delta k/k = s[f(k)/k] - (n+\delta) = \mathbf{(s/v^*) - (n+\delta)}$

Solow model

- Per capita capital, and so per capita GDP, will grow if
- $sf(k_t) > nk_{t+1} + \delta k_t$

- Per capita capital and per capita GDP will contract if

$$sf(k_t) < nk_{t+1} + \delta k_t$$

- Steady state equilibrium when everything grows at the same rate (time subscripts don't matter here)

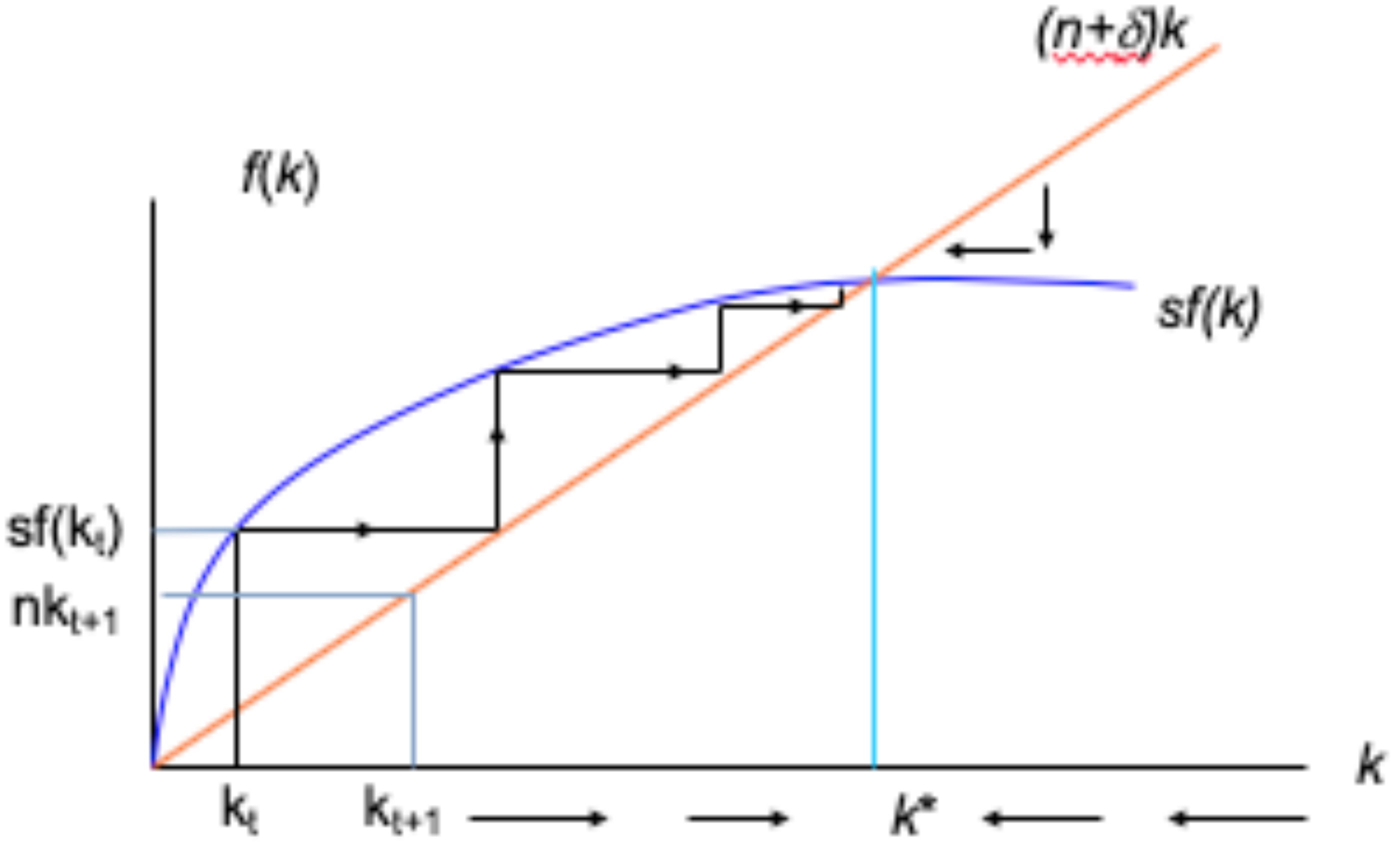
$\mathbf{sf(k^*) = (n+\delta)k^*}$ *or* **[**$s/v^* = n$, as $v^* = k^*/f(k^*)$**]**

Per capita capital and GDP will stop growing in the steady state equilibrium.

That is, GDP and population will grow at the same rate.

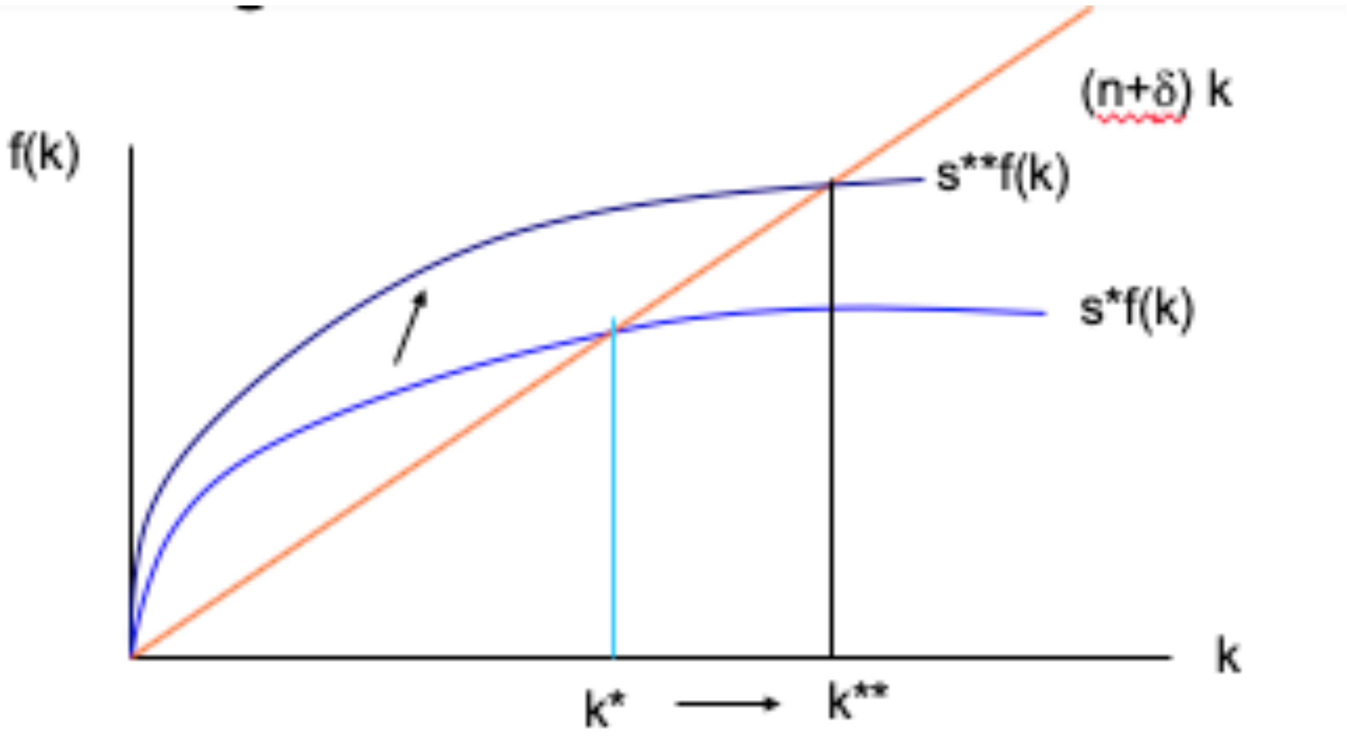
Graph

Solow growth model



Graph

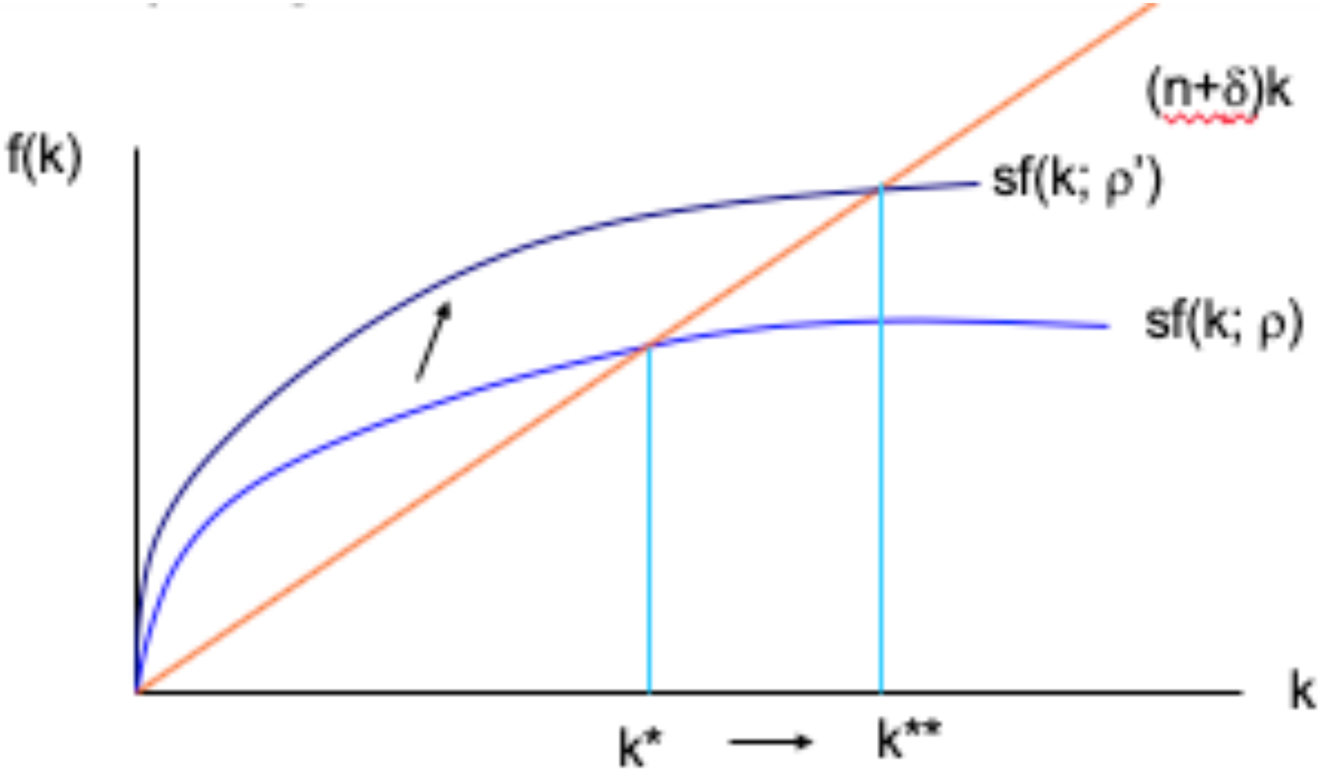
Higher savings rate



k*

Graph

- Technical progress (ρ)



Evaluation of the Solow model

- Empirically useful for growth accounting:
  - **Total factor productivity growth** (huge empirical literature)

- The long run prediction
  - Global **convergence** of per capita incomes
  - (Theoretically intriguing, but needs examination)

- Theoretical issue: **Sources of growth(?)**

- The factor productivity growth equation using Solow's growth model

Empirical usefulness

- The Solow model is useful in estimating technical progress, or productivity improvement.

- This is different from the growth of new jobs or capital goods. It is a component of GDP growth that is not accounted for by the growth of labour and capital use.

- GDP can grow if capital grows, and/or labour grows, and/or other input grows, and/or general productivity improves.

Empirical usefulness of the Solow model: Growth accounting

- Take a Cobb-Douglas production function:
- $Y_t = A_t K_t^{\alpha} L_t^{\beta}$, $0<\alpha, \beta<1$ →
- Growth rate of Y = **Growth rate of *A*** + α (growth rate of K use) + β (growth rate of L use)
- Object of interest: <u>Growth rate of *A*</u> (total factor productivity or TFP); this is estimated as a residual term, i.e. the **Solow residual**. → measuring <u>technical progress</u>

Derivation of the Solow growth accounting equation

- Write the Cobb-Douglas production function:
- $Y(t) = A(t)\, K(t)^{\alpha} L(t)^{\beta}$, $0<\alpha, \beta<1$,

- Take natural logarithm
- $\ln Y(t) = \ln A(t) + \alpha \ln K(t) + \beta \ln L(t)$
- Differentiate with respect to t:
- $\frac{dY}{dt}\frac{1}{Y} = \frac{dA}{dt}\frac{1}{A} + \alpha\frac{dK}{dt}\frac{1}{k} + \beta\frac{dL}{dt}\frac{1}{t}$
- $g_Y = g_A + \alpha g_K + \beta g_L$
- $g_A = g_Y - \alpha g_K - \beta g_L$ [TFP growth equation]

Total factor productivity growth equation

- **Growth rate of *A*** = Growth rate of *Y* - α (growth rate of *K* use) -β(growth rate of *L* use)
- There are no data on *A* or factor productivity.
- But there are data on Y (i.e. GDP), K, L and other inputs (if any). In addition a and b can be estimated by the regression method.
- So by using the Solow equation one can measure the growth of total factor productivity.

Solow model in practice

- Many authors applied this model to a number of countries, and East Asia in particular.
- It is believed that the East Asian miracle was due to significant productivity growth.
- Not only did employment grow significantly, but also the productivity of labour also grew during the East Asian industrialisation.

But did productivity really grow in East Asia?

- Economist Young (1995) debunked the East Asian miracle by showing that the East Asian growth was largely from increased capital use rather than from productivity growth

- Data for four Asian economies over 1966-90

	Output growth rate (%)	TFP growth rate (%)
Hong Kong	7.3	2.3
Singapore	8.7	0.2
South Korea	8.5	1.7
Taiwan	8.5	2.1

Solow model elsewhere: Senhadji (2000) for 1960-94

- Total factor productivity growth is small in the developing world.
- It is fallen in Africa.
- Contribution of human capital is highest in L America

	Output growth rate (%)	TFP growth rate (%)	Contribution of human capital to output growth (%)
East Asia	6.49	0.28	6.77
South Asia	4.66	0.55	5.36
Sub-Saharan Africa	2.83	-0.56	7.77
M.& Nrth Africa	5.05	-0.03	4.95
Latin America	3.42	-0.39	8.18

Long run prediction of the Solow growth model

- Zero growth of per capita income after a point, regardless of the country's initial condition, size or other characteristics.

Long run prediction of the Solow model: zero growth and **convergence** of per capita GDP

- In the long run all countries will reach the same steady state per capita GDP some day.
- This will happen regardless of the starting point (history does not matter for the long run per capita steady state GDP) → Unconditional (or absolute) convergence

For instance…

- The standard of Living in Japan, Taiwan or South Korea will not be any different to that of Western Europe.
- Their average income levels should also be similar (in PPP terms)
- Reason: Late starters will grow faster, because they don't need to 'reinvent the wheel', and they have access to more resources today than their predecessors had.

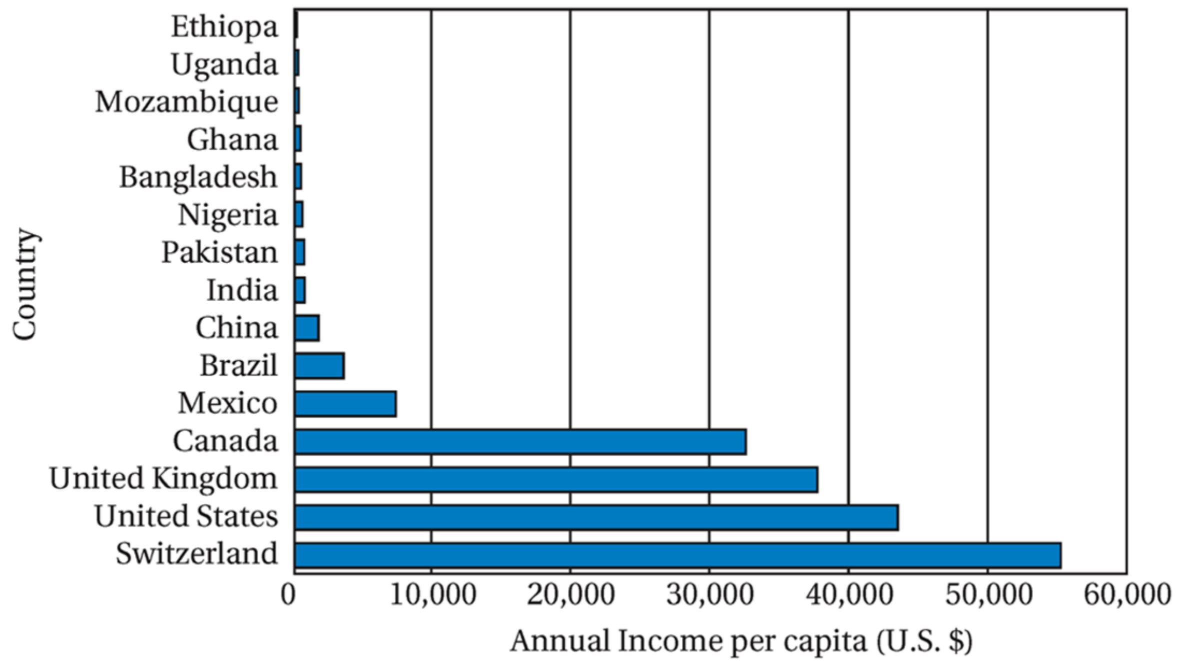
Late starter disadvantages

- But Kuznets argued that though developing countries have some late starter advantages (such as imitation, technological leapfrogging), they also face some greater challenges.
- <u>Today's developing countries differ in many ways</u> from the developed countries in their earlier stages : international migration, trade, R&D, domestic institutions.
- So it is unrealistic to expect that all developing countries will catch up with the developed countries at some point.

One piece of evidence: Divergence of income?

- Two centuries ago, the difference between the per capita incomes of the richest country and the poorest country was 3 to 1. Today it is 100 to 1.

- Is it divergence of income?



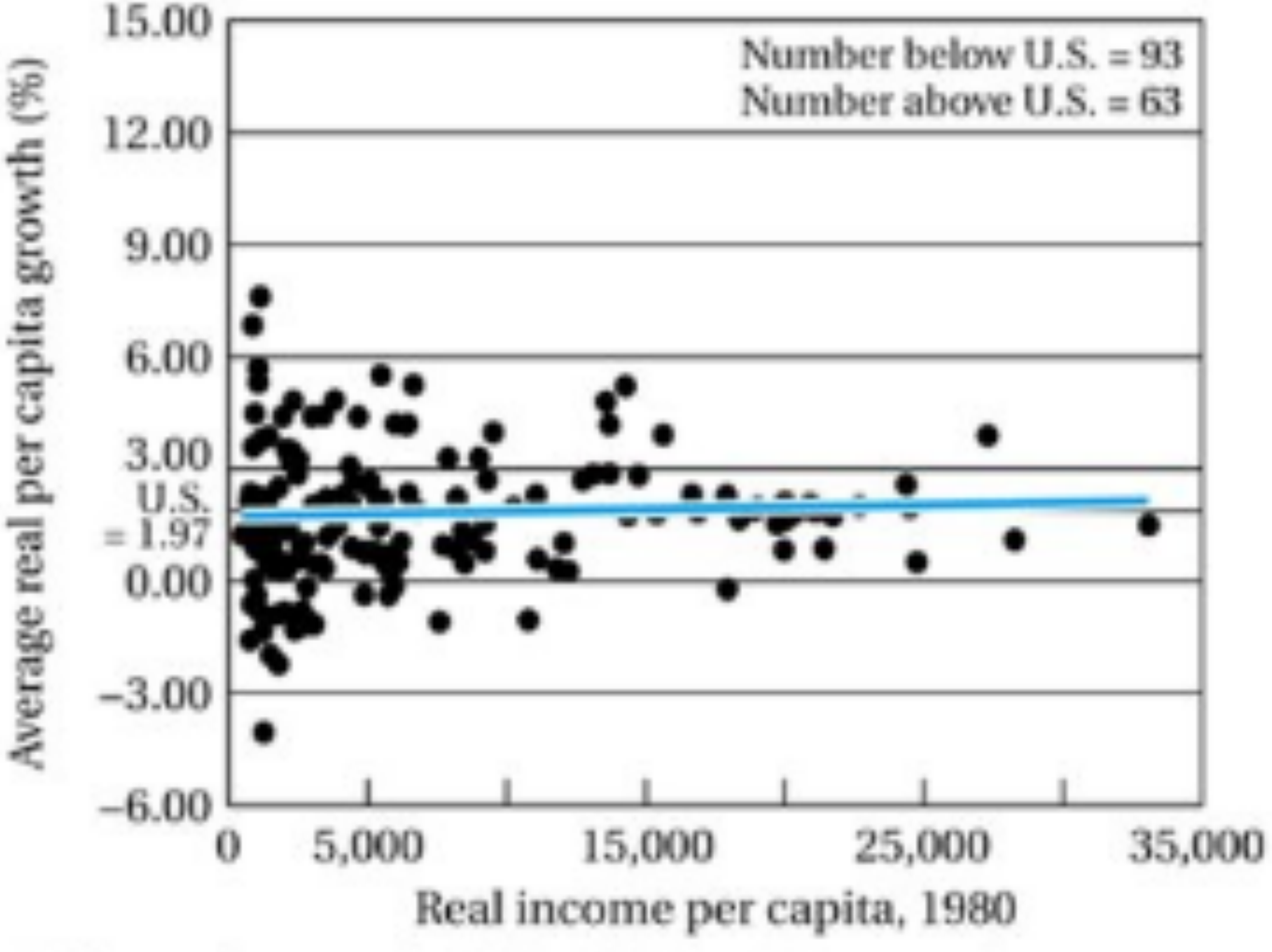
Source: Data from World Bank, *World Development Indicators, 2007* (Washington, D.C.: World Bank, 2007), tab. 1.1.

Testing of unconditional convergence or divergence

- Regress the following equation for a number of countries:
- $g = \alpha + \beta y_{baseline} + \varepsilon$
- Where g is the average growth rate of per capita GDP over a long time period (say at least for 30 years) and $y_{baseline}$ is the per capita GDP in the baseline year (say 1950)
- If $\beta < 0$ (and significant) unconditional convergence passes the test.
- Empirical finding of the majority of studies: No evidence for unconditional convergence, i.e. β is not significantly negative

Non-convergence

- When 157 developed and developing countries are studied over 1980-2007, there was no evidence of convergence of per capita income. Countries that had low income in 1980 are not growing faster.



(a) Per capita growth 1980–2007 for 157 countries

Conditional convergence: A modification of the idea of unconditional convergence

Assume countries are different in terms of *s*, *n* and various other factors (such as human capital)

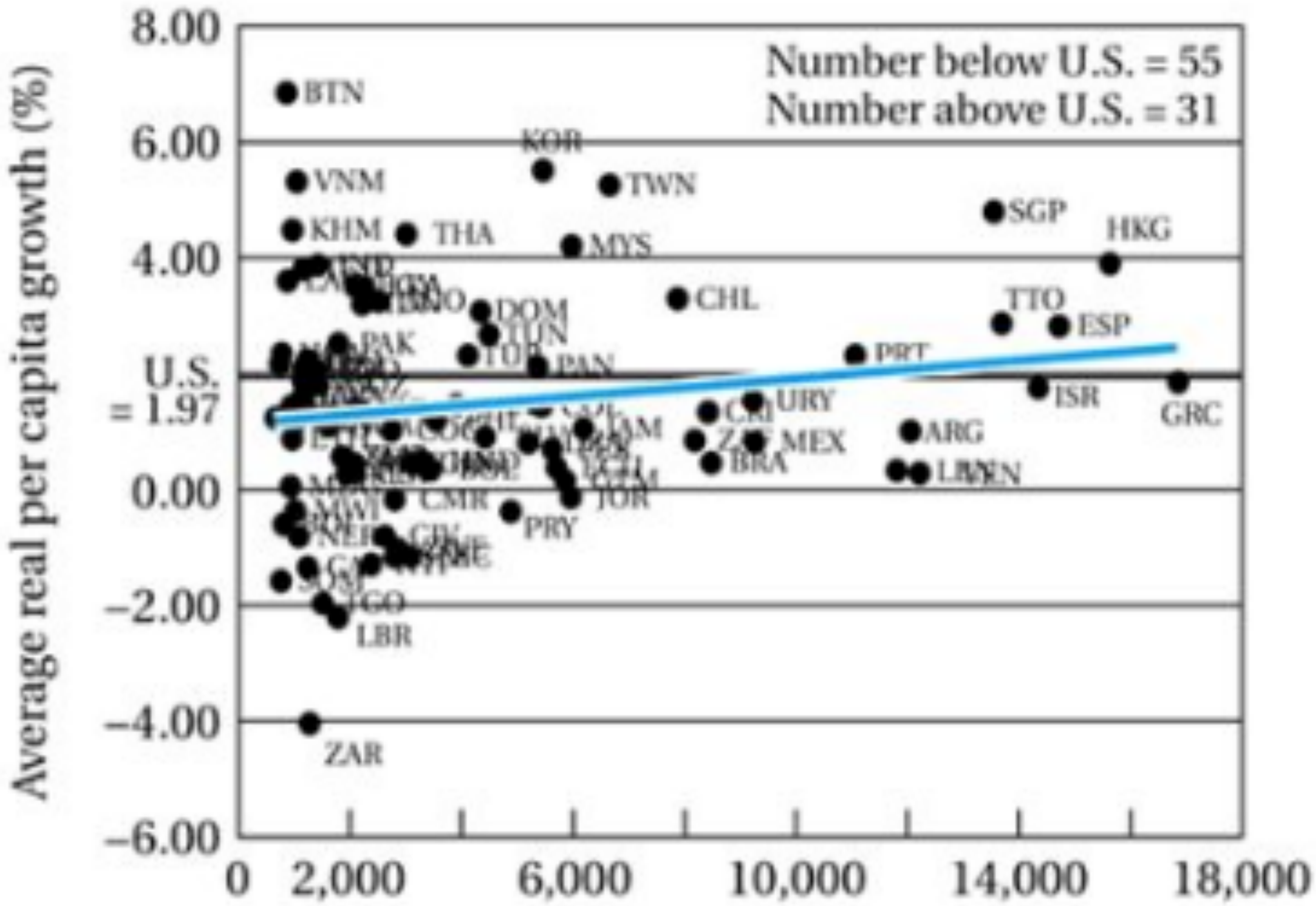
- Run the following regression now:

- $g = \alpha + \beta y_{baseline} + \gamma_1$ human capital + γ_2 resources + ε

- If $\beta < 0$ (and significant), then the model passes the test of <u>conditional convergence</u>.
- Countries that are similar (in other respects) may display the 'convergence'.
- Divergence may be due to dissimilarity in other factors.

Divergence among developing countries

- Evidence for conditional divergence among developing countries

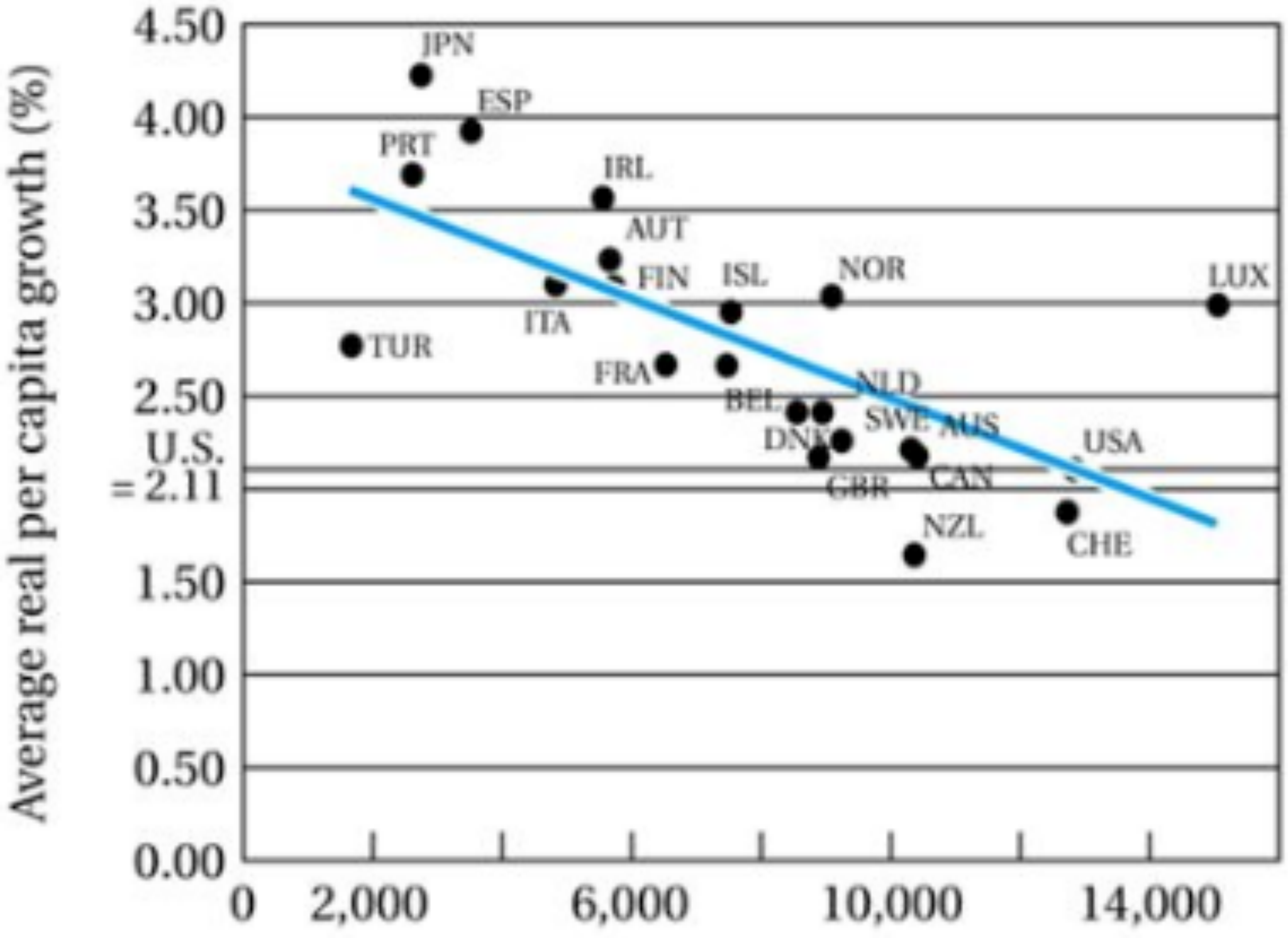


(b) Per capita growth 1980–2007 for 86 developing countries

- Source: Data from Center for International Comparisons, University of Pennsylvania

Convergence among developed countries

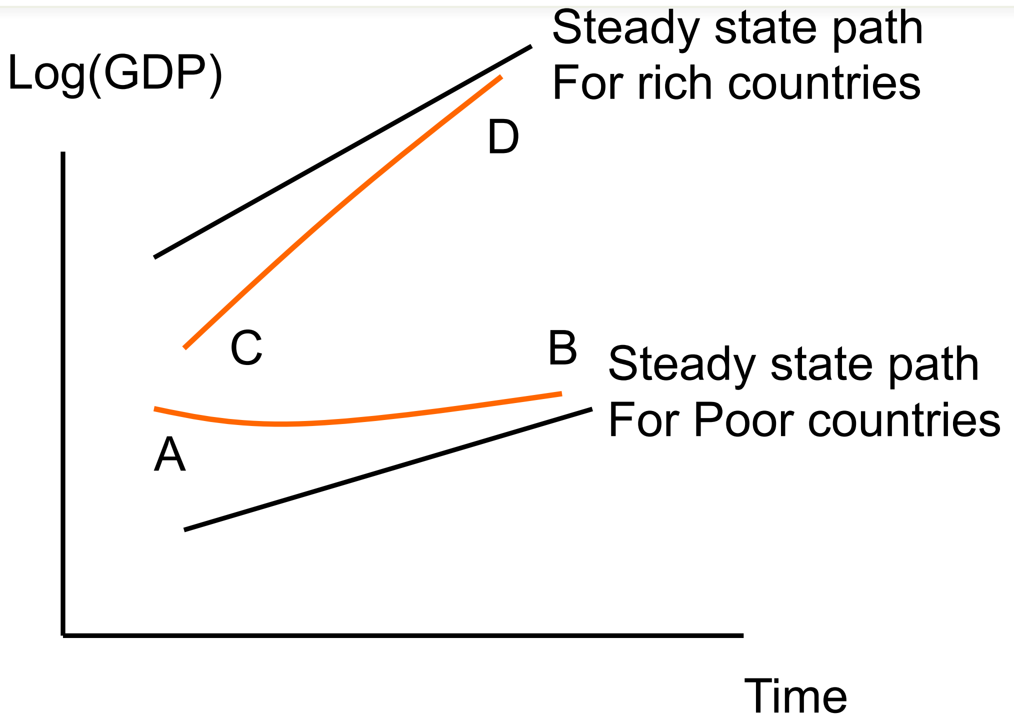
- 22 OECD countries Show convergence when their GDP growth data are plotted against their 1950 GDPs.
- Evidence for conditional convergence.
- Source: Data from Center for International Comparisons, University of Pennsylvania



(c) Per capita growth 1950–2007 for 22 OECD countries

Conditional convergence: Different destinies for different groups of countries

- Path AB: A poor country is converging to its steady state
- Path CD: A rich country is converging to its steady state.
- A poor country can have a lower (temporary) growth rate than a rich country.



Reconciling growth and divergence or conditional convergence

- But can we build a growth model that can demonstrate growth along with divergence or conditional convergence?
- Within the Solow model, we cannot do that.
- There are other theoretical difficulties

Theoretical difficulty: Sources of growth

Assumption of Diminishing returns to per capita capital: As saving is transformed into investment, its return in terms of next period's output growth diminishes, and eventually chokes off the growth

So growth stops after a point

- Only way growth can occur in the long run is via technical progress; but the model does not identify the source of technological progress

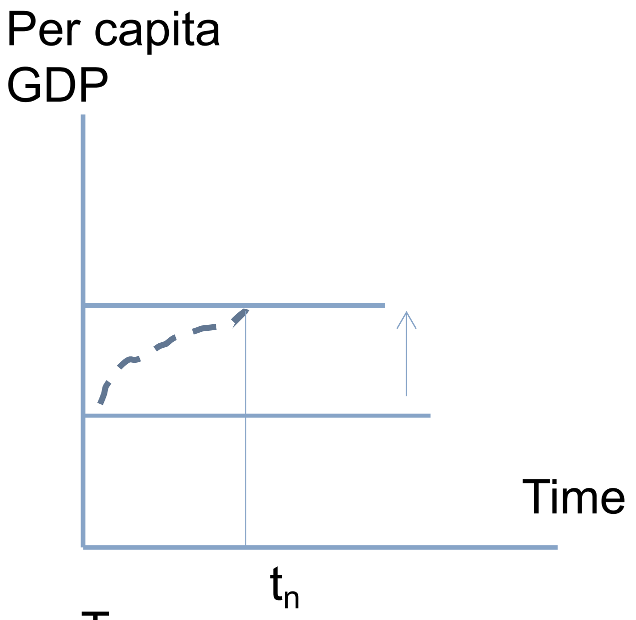
Theoretical difficulty: Sources of growth

- Other sources of growth are savings rate (*s*) and population growth rate (n), -- neither can be easily influenced by policy variables.

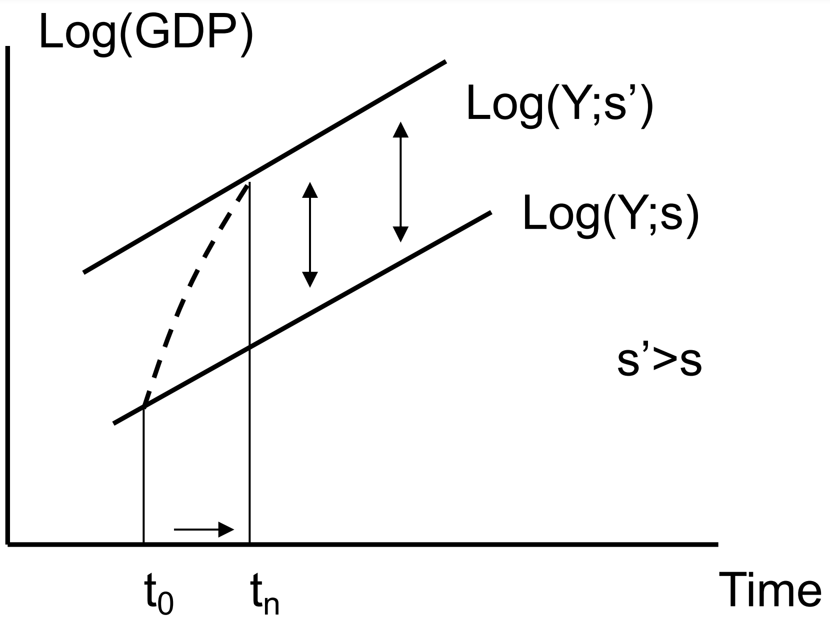
- In any case, technical progress, or an increase in *s* or a decrease in *n*, are not endogenous in the Solow model.

- So if these changes occur exogenously, then and only then growth can occur in the long run in the Solow model.
- **Thus, the Solow model is a model of <u>exogenous growth</u>.**

Solow: s has no long run growth effect: A change in s is exogenous and after a point its effect dies out.



Temporary growth in per capita Y



The saving rate improves the level of per capita GDP, but does not alter the growth rate of Per capita GDP

Endogenous Growth Theory

- Growth must be endogenous, meaning that the sources of growth must be accounted for within the model

Endogenous growth theory (1986, 1988 --)

- Tries to reconcile the predictions
  - of the Harrod-Domar model (positive long run growth) and
  - the Solow model (unconditional convergence)
  - and the empirical evidence of conditional convergence or diverse patterns of growth

- **Key question**: Where will the growth come from?

Endogenous growth theory (1986, 1988 --)

- So there has to be some escape route from diminishing returns that the Solow model suffers from.
- Endogenous growth theory: Two types of capital: physical capital and human capital.
- **Assumption:** diminishing returns to per capita capital, but increasing returns to human capital.
- It is the human capital that generates long run growth.

Sources of growth: Human capital in various forms, and its interactions with other factors

- Other Sources of growth in endogenous growth models:
  - Technical progress (via R&D and innovation)
  - Increasing returns to aggregate capital stock in the economy (technological spill-over, learning, network effect etc.)
  - Coordination and cooperation
  - Network effect: A firm's own acquisition of technology or capital enhances productivity of other firms which are using similar technology.

Generate any of these effects endogenously and this will help the economy grow on a sustained basis.

Some predictions of the new growth theories

- Conditional convergence ($\beta<0$) after controlling for initial low per capita income: poor countries have a tendency to grow faster.
- Conditional divergence ($\beta >0$) after controlling for initial level of low human capital: Poor countries tend to grow slower
- What is the intuition?

It all depends on the return to physical capital

Unskilled labour and low wage make capital an attractive input,
And thus greater use of capital generates faster growth

Low physical capital
relative to unskilled labour

(+)

Low per capita income;
This leads to less schooling

Ambiguous
Effect on rate
Of return to
Physical capital

Low human capital
Relative to unskilled
labour

(-)

But low human capital also tends to depress the returns to capital
And thus causes slower growth.

Two conflicting channels of returns to physical capital

- At an initial situation of low physical capital, capital's productivity will be high and so will be returns to it.

- But when physical capital is low, GDP is also low, human capital is also low, which reduces the returns to capital.

- If the first factor is dominant, we have conditional convergence.

- If the second factor is dominant we have conditional divergence.

Endogenous growth theory

- While the endogenous growth theory addresses most of the concerns raised by the empirical economists, it has become a technically complex area of study.

  - For development, we need to think beyond GDP growth.

  - But without growth development is nearly impossible.

The Romer model of endogenous growth

- Paul Romer provided a simple generalisation of the Solow model by
  - Introducing externality in aggregate capital
  - And abandoning CRS

- Each firm has the following technology
- $Y_i = AK_i^{\alpha} L_i^{(1-\alpha)} \overline{K}^{\beta}$ where $\overline{K}$ is the aggregate capital.

Externality is given by β.

- If all firms are identical, the aggregate production function will be

$$Y = AK^{\alpha+\beta} L^{(1-\alpha)}$$

[Note that $\overline{K}$ is simply replaced by K for simplicity]

Romer model

- Recall the investment equation:
- $\Delta K_t = I_t - \delta K_t$, I_t=S_t, and S_t=sY_t

- Then we have, $\Delta K_t = sY_t - \delta K_t$,
- $\frac{\Delta K_t}{K_t} = s\frac{Y_t}{K_t} - \delta$

- If $\frac{\Delta K_t}{K_t}$>0 and constant, say g, then $\frac{Y_t}{K_t}$ must also be constant.

- That means Y and K must grow at the same rate.

The Romer model: Growth rate of Y and K

- Consider the aggregate production function (with time subscript)
- $\Delta Y_t = MP_K \Delta K_t + MP_L \Delta L_t$
- Derive MP_K and MP_L from the Cobb-Douglas function:

The Romer model: Growth rate of Y and K

- $MP_K = A(\alpha + \beta)K^{\alpha+\beta-1}L^{1-\alpha}$

$$= A(\alpha + \beta)\frac{K^{\alpha+\beta}}{K}L^{1-\alpha} = (\alpha + \beta)\frac{Y}{K}$$

Likewise,

- $MP_L = A(1 - \alpha)K^{\alpha+\beta}L^{-\alpha}$

$$= A(1 - \alpha)\frac{L^{1-\alpha}}{L}K^{\alpha+\beta} = (1 - \alpha)\frac{Y}{L}$$

Romer model:

- Thus,
- $\Delta Y_t = MP_K \Delta K_t + MP_L \Delta L_t$
- $\Delta Y_t = Y_t[(\alpha + \beta)\frac{\Delta K_t}{K_t} + (1-\alpha)\frac{\Delta L_t}{L_t}]$
- $\frac{\Delta Y_t}{Y_t} = [(\alpha + \beta)\frac{\Delta K_t}{K_t} + (1-\alpha)\frac{\Delta L_t}{L_t}]$

- Note that $\frac{\Delta L_t}{L_t} = n$ and we want $\frac{\Delta Y_t}{Y_t} = \frac{\Delta K_t}{K_t} = g$.
- So, $g = (\alpha + \beta)g + (1-\alpha)n$.
- Or, $g = \frac{n(1-\alpha)}{(1-\alpha-\beta)}$.

Growth in per capita income

- Romer's distinguishing feature:
- Even though *Y/K* is constant (because Y and K are growing at the same rate), *Y/L* is not.
- That is, $g>n$:
- Growth in *Y/L*:
  - $g - n = \frac{n(1-\alpha)}{(1-\alpha-\beta)} - n = \frac{\beta n}{(1-\alpha-\beta)}$

  - Growth is coming from the externality term β.

Romer model

- Finally, recall the investment equation
- $\frac{\Delta K_t}{K_t} = s\,\frac{Y_t}{K_t} - \delta$
- Substitute *g* in the LHS:
- $\frac{n(1-\alpha)}{(1-\alpha-\beta)} = s\,\frac{Y_t}{K_t} - \delta$

- Next, write Y_t/K_t as $(Y_t/L_t)/(K_t/L_t)$ [in per capita term]
- $\frac{n(1-\alpha)}{(1-\alpha-\beta)} = s\,\frac{y_t}{k_t} - \delta$

- Subtract n from both sides:
- $\frac{n(1-\alpha)}{(1-\alpha-\beta)} - n = s\,\frac{y_t}{k_t} - \delta - n$
- Or, $\frac{n\beta}{(1-\alpha-\beta)} + n + \delta = s\,\frac{y_t}{k_t}$
- Or, $sy_t = k_t[n\left(\frac{\beta}{1-\alpha-\beta} + 1\right) + \delta]$ [In equilibrium]

Romer model

- The Romer growth equation
- $sy_t = k_t[n\left(\frac{\beta}{1-\alpha-\beta} + 1\right) + \delta]$

- If β=0, we return to the Solow model.
- With β>0, (provided α+β < 1), g>n, and hence per capita income will grow indefinitely.

Romer model

- The Romer growth equation
- $sy_t = k_t[n\left(\frac{\beta}{1-\alpha-\beta} + 1\right) + \delta]$

- Note that in the Romer model, *K* and *Y* will grow at a higher rate than *L*. So *y* and *k* will not be constant.
- *K* and *Y* grow at the constant rate g. So one can write them as, $K_t = K_0 e^{gt}$, $Y_t = Y_0 e^{gt}$ and for L we have $L_t = L_0 e^{nt}$

- So, $s\frac{Y_t}{L_t} = \frac{K_t}{L_t}\left[n\left(\frac{\beta}{1-\alpha-\beta} + 1\right) + \delta\right]$

Romer growth equation

- Note that in the Romer model, *K* and *Y* will grow at a higher rate than *L*. So *y* and *k* will not be constant.
- *K* and *Y* grow at the constant rate g. So one can write them as, $K_t = K_0 e^{gt}$, $Y_t = Y_0 e^{gt}$ and for L we have $L_t = L_0 e^{nt}$
- $s\frac{Y_0 e^{gt}}{L_0 e^{nt}} = \frac{K_0 e^{gt}}{L_0 e^{nt}}[n\left(\frac{\beta}{1-\alpha-\beta} + 1\right) + \delta]$
- Or, by cancelling the common terms from both sides,

Romer growth equation

$$s\,\frac{Y_0}{L_0} = \frac{K_0}{L_0}[n\left(\frac{\beta}{1-\alpha-\beta}+1\right)+\delta]$$

- $sy_0 = k_0[n\left(\frac{\beta}{1-\alpha-\beta}+1\right)+\delta]$

- Where $y_0 = \frac{Y_0}{L_0}$ and $k_0 = \frac{K_0}{L_0}$ are the initial level of per capita GDP and per capita capital.