A note on Piketty and diminishing returns to capital

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Abstract

Capital in the Twenty-First Century predicts a rise in capital's share of income and the gap $r - g$ between capital returns and growth. In this note, I argue that neither outcome is likely given realistically diminishing returns to capital accumulation. Instead—all else equal—more capital will erode the economywide return on capital. When converted from gross to net terms, standard empirical estimates of the elasticity of substitution between capital and labor are well below those assumed in *Capital*. Piketty (2014)'s inference of a high elasticity from time series is unsound, assuming a constant real price of capital despite the dominant role of rising prices in pushing up the capital/income ratio. Recent trends in both capital wealth and income are driven almost entirely by housing, with underlying mechanisms quite different from those emphasized in *Capital*.

1 Introduction

Capital in the Twenty-First Century is a work of remarkable scope and influence, with a sweeping new view of income, wealth, and inequality. It builds on a singular trove of data assembled by Piketty and coauthors, and it is sure to be a centerpiece of the debate for years to come. Although the book is widely recognized for its empirical contributions, it also uses this data to construct a distinctive theory about the trajectory of the wealth and income distribution.

One of its central themes is a story of capital accumulation. In Piketty (2014)'s framework, slower growth will produce a rise in the ratio of capital to income. This, in turn, will bring about an expansion in capital's share of income. Meanwhile, as the growth rate g dwindles and the return on capital r holds relatively steady, the gap $r - g$ will expand, allowing existing accumulations of wealth to grow more rapidly relative to the economy as a whole. This will aggravate inequality in the wealth distribution. In short, a key message of *Capital in the Twenty-First Century* is that capital's role in the economy will grow in the twenty-first century.

*Please send any comments or corrections to mrognlie@mit.edu. Special thanks are due to Thomas Piketty and Gabriel Zucman for making their data open and easily accessible.

As Piketty (2014) readily acknowledges, diminishing returns may be problematic for this thesis. If the return on capital falls quickly enough when more capital is accumulated, capital's share of income will fall rather than rise—so that even as the balance sheets of capital owners expand, their claim on aggregate output will shrink. Furthermore, with a sufficient decline in r, as g falls the gap $r - g$ will narrow as well. But Piketty (2014) argues that diminishing returns, although undoubtedly present, are unlikely to be so strong; this view is offered in additional depth in a companion journal article, Piketty and Zucman (2013).

This note articulates the opposite view: most evidence suggests diminishing returns powerful enough that further capital accumulation will cause a decline in net capital income, rather than an expansion. If, following Piketty (2014)'s model of savings, a decline in g causes an expansion in the long-term capital stock, both the net capital share of income and $r - g$ are likely to decline. These conclusions are not definite—there are many obstacles to empirical certainty here—but they do counsel skepticism about Piketty (2014)'s central outlook.

In Section 2, I discuss the economic concept central to diminishing returns, the elasticity of substitution between capital and labor. When this elasticity is greater than one, a higher capital/income ratio is associated with a higher share of capital income; when the elasticity is less than one, the opposite is true. A crucial detail is whether this elasticity is defined in gross or net terms; net subtracts depreciation from income, while gross does not. Although Piketty (2014) and Piketty and Zucman (2013) rightly affirm that net concepts are more relevant for an analysis of inequality, they do not cover the distinction between net and gross elasticities. This is problematic, because net elasticities are mechanically much lower than gross ones, and the relevant empirical literature uses gross concepts. The vast majority of estimates in this literature, in fact, imply net elasticities less than 1—well below the levels needed by Piketty (2014).

Using elasticities within the standard range of empirical estimates, I show that an increase in the capital/income ratio is more likely to be associated with substantial decrease in capital's net share of income, not an increase. $r - g$ is likely to fall as well. These calculations are backed by a simple intuition: the net return r on capital equals the gross return R minus the depreciation rate δ. As capital rises and diminishing returns kick in, R will fall while δ remains constant, compressing the net return $r = R - \delta$ by a greater proportion than the gross return R. The hit from diminishing returns is always more painful in net terms.

In Section 3, I cover the time series evidence marshaled by Piketty and Zucman (2013), and cited by Piketty (2014), in support of the claim that the net elasticity of substitution is above 1. Their story is simple: across a sample of eight developed economies, both the capital income ratio and capital's net share of income have risen substantially in the last several decades. Under certain assumptions—particularly the assumption that the real price of capital is constant—this is only consistent with a net elasticity greater than 1. When these assumptions are violated, however, inference becomes much more difficult. Indeed, changes in the real price of capital tend to induce comovement in the two series examined by Piketty and Zucman (2013), for *any* value of the net elasticity. Empirically, this is a first-order concern: in the paper's data decomposition, once the contribution from changes in real capital prices is taken out, nearly the entire increase in the average capital/income ratio disappears.

Also using Piketty and Zucman (2013)'s data, I find that a single component of the capital stock—housing—accounts for nearly 100% of the long-term increase in the capital/income ratio, and more than 100% of the long-term increase in the net capital share of income. In other words, when housing is removed, the long-term comovement emphasized by Piketty and Zucman (2013) as evidence for a high net elasticity no longer holds—there is instead a small increase in the capital/income ratio and a small decrease in the net capital share of income. Although this alone is not a reason to exclude housing, closer analysis reveals that housing alone probably cannot be responsible for a high aggregate elasticity of substitution. If housing and other forms of consumption were highly substitutable, for instance, a rise in real rental costs would be associated with a decline in the housing share of expenditure—but exactly the opposite appears true in practice. More likely, the dominant role of housing in Piketty and Zucman (2013)'s data reflects the influence of real price changes, particularly for land. And while this is an important story, it does little to support Piketty (2014)'s claim of weak diminishing returns.

Section 4 is more speculative. It reflects on a common story about technology and capital—that as we develop new and innovative ways to use capital, diminishing returns will matter less and less. I observe that the vast majority of the value of the existing capital stock is in structures—houses, apartments, and offices—rather than equipment or intellectual property. Advanced technology accounts for only a small fraction of the capital stock, and this fraction has been roughly stable over the last several decades. While it has played a crucial role in recent economic growth, tech-intensive capital is not large enough at current prices to absorb much of the substantial increase envisioned by Piketty (2014) in the value of the capital stock. By comparison, structures—which depreciate less than equipment, and are accordingly more sensitive to the net cost of capital—play an outsize role in absorbing aggregate savings. I illustrate this in a stylized model where capital is split between two sectors.

Section 5 concludes.

2 Elasticity of substitution

2.1 Overview

Will a rise in the capital stock translate into a higher share of income going to owners of capital? It turns out that this question is ambiguous: the answer may depend on whether we care about *gross* or *net* income. Net income subtracts capital depreciation, while gross income does not. Both measures can be useful, but when studying distribution it generally makes sense to use net income: if someone earns $1 in revenue from renting out a building but loses $0.40 as the building deteriorates, her command over resources has only increased by $0.60. (This is why, for instance, corporate profits are generally measured net of depreciation.)

In light of this observation, both Piketty (2014) and Piketty and Zucman (2013) are careful to use only net concepts. Although their treatment is internally consistent, the distinction between net and gross is critical when comparing their conjectures to other estimates, which are often stated in gross terms. Indeed, as we will see, their basic story only works under parameter values far out of line with estimates in the prior literature.

2.2 Elasticity of substitution basics

Let K denote capital and L denote labor, and let $F(K, L)$ denote production as a function of these two inputs. Suppose that F has constant returns to scale and has positive but diminishing marginal returns in each factor.

The **elasticity of substitution** σ between capital K and labor L is classically defined to be

$$\sigma \equiv \frac{F_K \cdot F_L}{F \cdot F_{KL}} \tag{1}$$

This gives us the (inverse) response of the ratio F_K/F_L of marginal products to the ratio K/L of capital: $d(\log(F_K/F_L))/d(\log(K/L)) = -1/\sigma$. Equivalently, σ tells us the extent to which the producer's relative demand for K/L will change if there is a change in the relative cost R/W of using capital and labor as inputs.

The elasticity of F_K with respect to a change in the capital-output ratio K/F is also given by $-1/\sigma$. This implies that the elasticity of the capital income *share* $F_K K/F$ with respect to the capital-output ratio K/F is

$$\frac{d(\log(F_K K/F))}{d(\log(K/F))} = 1 - \frac{1}{\sigma} \tag{2}$$

This indicates the critical importance of the threshold $\sigma = 1$. If $\sigma > 1$, the elasticity is positive, so that the capital income share will increase as K/F rises. Inversely, if $\sigma < 1$, the capital income share will fall as K/F rises. In the important special case $\sigma = 1$, diminishing returns exactly offset the increased quantity of capital, and the share remains constant.

2.3 Net vs. gross

Thus far, I have been ambiguous about whether the function F gives *gross* production, or production *net* of capital depreciation.

In principle, either interpretation is legitimate. If F is *gross* production, then $1 - 1/\sigma$ is the elasticity of *gross* capital income with respect to the ratio of capital to *gross* output. If F is *net* production, then $1 - 1/\sigma$ is the elasticity of *net* capital income with respect to the ratio of capital to *net* output. Piketty (2014) consistently uses the net interpretation, which is more meaningful when studying income distribution. It is important to recognize, however, that σ depends greatly on which measure is used.

Suppose that $Y = F(K, L)$ is the gross production function, and that its elasticity of substitution is σ. Then the net production function is $\tilde{F}(K, L) = F(K, L) - \delta K$, and recalling (1) its elasticity is

$$\begin{aligned} \frac{\tilde{F}_K \tilde{F}_L}{\tilde{F}_{KL}\tilde{F}} &= \frac{(F_K - \delta)F_L}{F_{KL}(F - \delta K)} \\ &= \underbrace{\frac{F_K F_L}{F_{KL} F}}_{\sigma} \cdot \frac{F_K - \delta}{F_K} \cdot \frac{F}{F - \delta K} \end{aligned} \tag{3}$$

Thus the elasticity of substitution for the net production function ("net elasticity") equals the elasticity of substitution for the gross production function ("gross elasticity") times

(A) the ratio $(F_K - \delta)/F_K$ of the net and gross returns from capital, and

(B) the ratio $F/(F - \delta K)$ of gross and net output.

The ratio in (A) is below 1, while the ratio in (B) is above 1. Critically, the product of these ratios is always less than 1, so that the net elasticity is always below the gross elasticity. This observation is simple: $(F_K - \delta)/F_K$ is the ratio of net to gross capital income, while $(F - \delta K)/F$ is the ratio of net to gross total income. Since there is no distinction between net and gross for labor income, the former ratio is smaller than the latter.[1]

Intuitively, the net elasticity is smaller than the gross elasticity because for a given decline in the the gross return F_K on capital, the proportional impact on the net capital return $F_K - \delta$ is larger. If $F_K = 0.10$ and $\delta = 0.05$, then a 10% decline in F_K from 0.10 to 0.09 implies a 20% decline in $F_K - \delta$ from 0.05 to 0.04. When gross returns decline but depreciation does not, net returns decline even more rapidly.

A rough calibration. According to the BEA, in the US the depreciation share of gross output is 15.8%, implying that $F/(F - \delta K) \approx 1.19$.[2] Meanwhile, the average depreciation rate of private fixed assets is 5.7%.[3] Since the arithmetic average is not exactly appropriate

[1] Explicitly, $\frac{F_K - \delta}{F_K} = \frac{F_K K - \delta K}{F_K K} < \frac{F_K K + F_L L - \delta K}{F_K K + F_L L} = \frac{F - \delta K}{F}$

[2] This is line 5, consumption of fixed capital, divided by line 1, gross domestic product, for 2013 in NIPA Table 1.7.5: $2646.6/16799.7 \approx 15.8\%$.

[3] This is line 1, depreciation of private fixed assets at current cost, of Table 2.4 divided by line 1, current cost net stock of private fixed assets, of Table 2.1 in the BEA's fixed assets accounts, for 2012: $2049.2/36215.6 \approx 5.7\%$.

when aggregating heterogenous capital, to be conservative I will use a depreciation rate of only $\delta = 0.04$. An appropriate level for the net return on capital $F_K - \delta$ is difficult to determine, but in line with the 5% figure commonly used in Piketty (2014) I will use $F_K - \delta = 0.05$.[4]

Given these figures, the net elasticity is lower than the gross elasticity by a factor of

$$\frac{F_K - \delta}{F_K} \cdot \frac{F}{F - \delta K} \approx \frac{0.05}{0.09} \cdot 1.19 \approx 0.66$$

Thus a net elasticity of 1.5, for instance, requires a gross elasticity of roughly 2.27. This disparity generally becomes larger, however, as K increases and F_K declines, bringing down the ratio $(F_K - \delta)/F_K$ of net to gross capital income.[5] Comparing elasticities at current levels of K therefore understates the distinction between gross and net for large increases in capital, and it is informative to examine the consequences of a large increase directly. This is where I turn next.

Some quantitative illustration. Continuing to assume initial levels of $F/(F - \delta K) = 1.19$, $\delta = 0.04$ and $r = F_K - \delta = 0.05$, Figure 1 shows how the net capital share of income rK/Y^{net} changes as we increase the ratio K/Y^{net} of capital to net income, under various assumptions about the gross elasticity of substitution σ^{gross}. (Changes are stated as ratios, so that '1' implies no change, '1.5' implies a 50% increase, and '2' implies doubling. The precise calculation is described in Appendix B.1.)

Notably, when $\sigma^{gross} = 0.5$, the net capital share actually becomes negative when the capital-net income ratio doubles, as the gross return to capital falls below the depreciation rate. In fact, this eventually happens for any constant level of σ^{gross}—the gross return to capital becomes arbitrarily small as the capital/output ratio rises, implying that the net return eventually becomes negative. This is, however, not visible in Figure 1 for any of our alternative choices of σ^{gross}, since it only kicks in for even larger increases in K/Y^{net}.

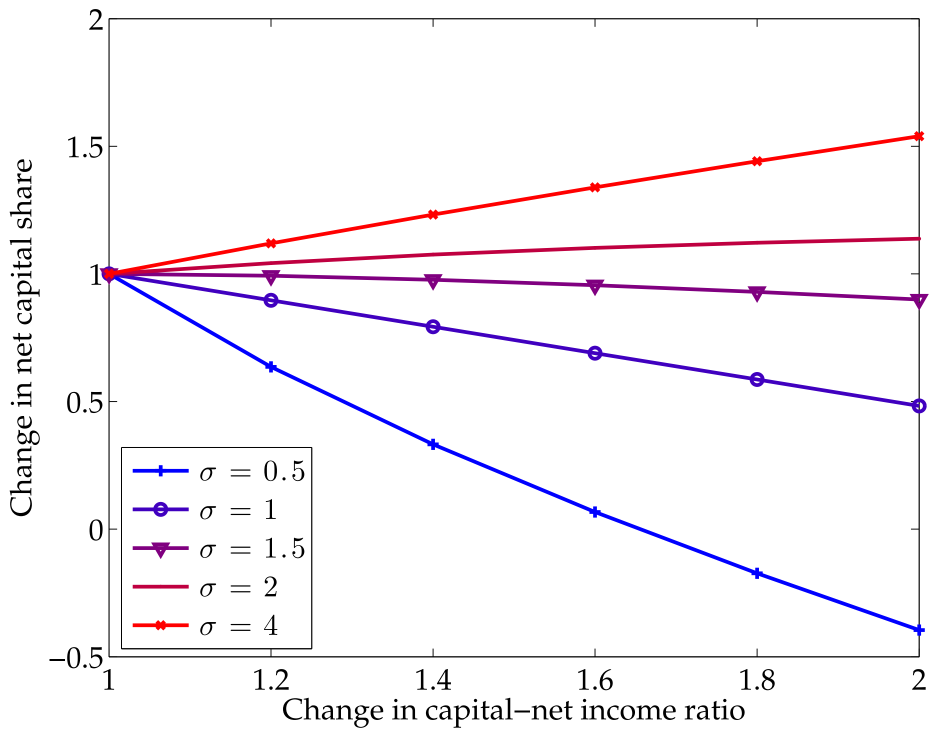
When $\sigma^{gross} = 1$, the net capital share falls in half as the capital-net income ratio doubles—a remarkable contrast with $\sigma^{net} = 1$, which implies a constant net share. ($\sigma^{gross} = 1$ implies that the gross capital share is constant, but as the gross return falls, more and more of this is eaten up by depreciation.) With $\sigma^{gross} = 1.5$ and $\sigma^{gross} = 2$, the net capital share declines slightly and increases slightly, respectively. Only $\sigma^{gross} = 4$ yields a robust increase, of slightly more than 50%, in the net capital share in response to a doubling in the capital-net income ratio.

I now turn to the empirical plausibility of these values.

[4] If we simply divide net capital income measured in the national accounts by the stock of private fixed assets, we obtain a much higher number. This is because much of the measured net return on capital consists of land rents, monopoly rents, and returns on intangible capital not measured in the fixed assets accounts. Since land and monopoly rents are not produced by accumulable capital, and thus are not directly relevant to the Piketty (2014) story of greater capital accumulation leading to higher net capital income, I will not focus on them at the moment. Unmeasured capital is difficult because its depreciation is not recorded, meaning that any attempt to include the returns from unmeasured capital while not also including an estimate for depreciation will bias downward the depreciation share of gross capital income.

[5] This is true unless the elasticity is so high that $(F_K - \delta)/F_K$ falls by less than $(F - \delta K)/F$.

Figure 1: Relationship between net capital-income ratio and net capital share, by gross elasticity σ.



2.4 Empirical literature on the elasticity of substitution

Ever since Arrow, Chenery, Minhas and Solow (1961) first proposed the constant elasticity of substitution (CES) production function, researchers have attempted to estimate the key elasticity parameter σ. These studies have virtually always looked at the elasticity of substitution in the *gross* production function.

The literature is vast and its conclusions muddled, but one consistent theme has been the rarity of high elasticity estimates. Chirinko (2008) provides an excellent summary of the empirical literature, listing estimates from many different sources and empirical strategies. Of the 31 sources[6] listed for the gross elasticity, fully 30 out of 31 show $\sigma^{gross} < 2$. 29 out of 31 show $\sigma^{gross} < 1.5$, and 26 out of 31 show $\sigma^{gross} < 1$. The median is $\sigma^{gross} = 0.52$, and Chirinko concludes that "the weight of the evidence suggests that σ^{gross} lies in the range between 0.40 and 0.60". Table 1 shows the full distribution.

Figure 1 shows the implications of this rough consensus value of $\sigma^{gross} = 0.5$. As discussed above, given this value even small increases in the capital-output ratio K/Y^{net} lead to a plummeting net capital share. Even with $\sigma^{gross} = 1$, which is higher than over 80% of the estimates listed in Chirinko (2008), Figure 1 indicates that the net capital share falls substantially when the capital-output ratio rises. A gross elasticity of $\sigma^{gross} = 2$,

[6] For a few sources that list a range of elasticities, I take the midpoint. This has minimal effect on the distribution.

Table 1: Distribution of elasticity estimates compiled by Chirinko (2008).

σ	$[0, 0.5)$	$[0.5, 1)$	$[1, 1.5)$	$[1.5, 2)$	$[2, 4)$
Frequency	14	12	3	1	1

which is needed to obtain even a mild increase in the net capital share, is an extreme outlier in Table 1, above 30 out of 31 estimates. A gross elasticity of $\sigma^{gross} = 4$, needed to obtain a large increase in the net capital share, is outside the range of Table 1 altogether.

Clearly, the literature is imperfect and inconclusive, and not all estimates directly correspond to the aggregate elasticity that interests us. Furthermore, if some substitution between labor and capital only takes place in the very long term, it is possible that the studies in Chirinko (2008) systematically understate the true long-run elasticity. Nevertheless, it is important to emphasize that the σ^{gross} required by the story in Piketty (2014)—where capital accumulation leads to substantially greater net capital income—is far outside the range of values that most economists studying this issue believe empirically plausible. Given the consensus view, Piketty (2014)'s forces are in fact more likely to result in a *decrease* in net capital income.

2.5 Implications for $r - g$

One of the main themes in Piketty (2014) is the gap $r - g$ between the real return r on capital and the real growth rate g of the economy. This gap, for instance, gives the rate at which a wealthy dynasty can withdraw capital income for consumption purposes without decreasing its wealth relative to the size of the economy. More generally, when $r - g$ is higher, "old" accumulations of wealth become more important relative to "new" ones. Higher $r - g$ generally implies that the power law tail of the wealth distribution has a smaller exponent—so that there is more inequality of wealth at the top, and extreme levels are more likely. Many readers take the dynamics of $r - g$ to be *the* central theme of the book.

The basic argument in Piketty (2014) to support an increase in $r - g$ is that as g declines (due to slower population growth, and possibly a productivity growth slowdown), r is unlikely to fall by the same amount. One possible supporting argument is that historically, r has stayed relatively constant even amid large changes in g. (See footnote 10 for further discussion of this argument.) For now, we can investigate the elasticities that would be necessary for this conclusion to hold, given Piketty (2014)'s other assumptions.

Both Piketty (2014) and Piketty and Zucman (2013) make heavy use of the identity

$$\frac{K}{Y^{net}} = \frac{s}{g} \tag{4}$$

where s is the net savings rate and g is the growth rate, which is dubbed the "Second Fundamental Law of Capitalism". This identity only holds asymptotically—if s or g changes, convergence to the new value of K/Y^{net} does not happen instantaneously—and it is unlikely that s is exogenous and invariant to changes in g. Nevertheless, Piketty (2014) argues that it is useful to explore the implications of this identity given exogenous s, particularly the fact that K/Y^{net} rises as g falls.[7] Based on this observation, it claims:

> The return to a structurally high capital/income ratio in the twenty-first century, close to the levels observed in the eighteenth and nineteenth centuries, can therefore be explained by the return to a slow-growth regime. Decreased growth–especially demographic growth–is thus responsible for capital's comeback.

Adopting Piketty (2014)'s approach of assuming that s is invariant to g, we can calculate the asymptotic behavior of $r - g$ as g decreases under various assumptions about the elasticity of substitution.[8] Suppose that we start with $r = 5\%$ and $g = 3\%$, and continue to use the parameters underlying Figure 1.

Figure 2 shows the $r - g$ that results as g falls below 3% and $K/Y^{net} = s/g$ rises. Notably, the gap $r - g$ falls dramatically from its original value of 2.0% for all σ^{gross} below 4. For $\sigma^{gross} = 4$, $r - g$ increases slightly but starts decreasing at $g = 0.5\%$, with a maximum of around $r - g \equiv 2.35\%$. Only $\sigma^{gross} = 8$ displays a robust increase. Recall that both $\sigma^{gross} = 4$ and $\sigma^{gross} = 8$ are higher than all 31 estimates from Chirinko (2008) discussed in Section 2.4.

Given Piketty (2014)'s assumption that $K/Y^{net} = s/g$ with fixed s, it appears virtually impossible to generate a substantial increase in $r - g$ in response to a decline in g. For empirically plausible values of the elasticity, the diminishing returns induced by higher K/Y^{net} cause r to drop by at least as much as g.

Piketty (2014)'s projections to the contrary implicitly rely on extreme values for the elasticity. Figure 10.9 in Piketty (2014), which displays the historical time series of r and g, projects a decline of from the 1950–2012 average values of $r = 5.3\%$ and $g = 3.8\%$ to values in 2050–2100 of $r = 4.3\%$ and $g = 1.5\%$. Continuing to assume $K/Y^{net} = s/g$, this requires a net elasticity of

$$\frac{\log(3.8/1.5)}{\log(5.3/4.3)} \approx 4.4$$

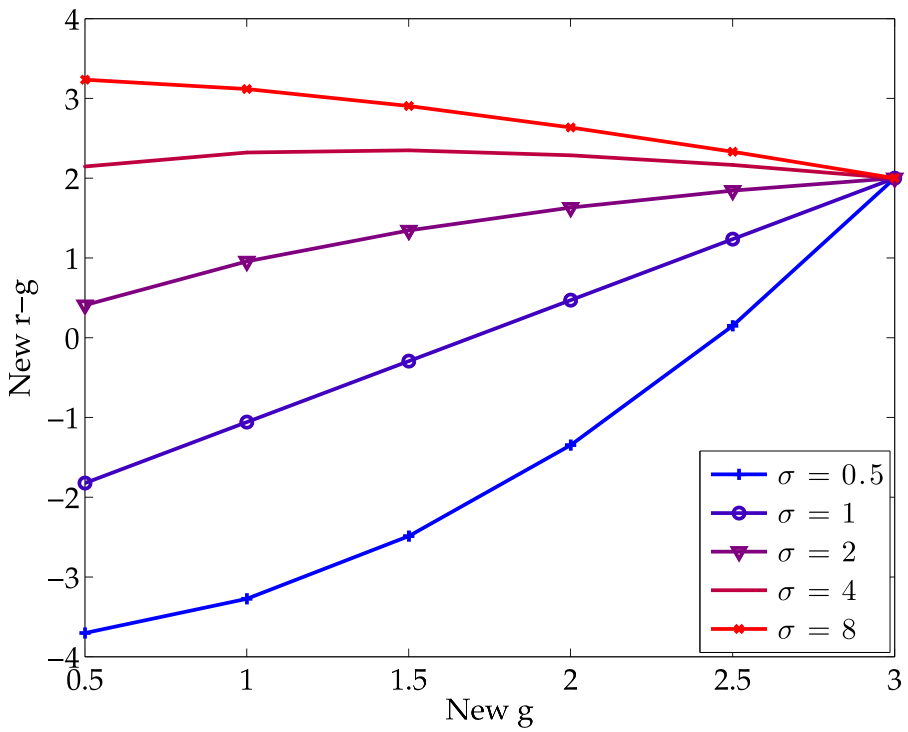
which is equivalent to an even higher gross elasticity, again well outside the range of estimates cited by Chirinko (2008) in Table 1.

Alternative hypotheses on savings. One way to make increases in $r - g$ more plausible is to dispose with Piketty (2014)'s assumption of exogenous s in the long-run identity $K/Y^{net} = s/g$. Economic models generally endogenize s as the optimal decision of

[7] There is some conflict between the assumption of exogenous s for all income and the emphasis on $r - g$. If only this fraction s of capital income r is saved, then existing fortunes will grow at the rate $s \cdot r - g$, not $r - g$; and for plausible values of s as a share of all income, $s \cdot r - g$ is likely to be quite negative, implying the rapid erosion of existing wealth. We can avoid this implication by saying that the relevant s for capital income is higher than s for labor income, which also implies that the aggregate savings rate will change as the mix of labor and capital income changes. Since this approach is not pursued in Piketty (2014), I will not complicate matters by pursuing it here.

[8] Given (4), K/Y^{net} is inversely proportional to g. Given the change in K/Y^{net} and the gross elasticity σ, we can calculate the implied change in r in the same way as for Figure 1. See Appendix 3 for the exact calculation.

Figure 2: Asymptotic $r - g$ following change in g, by gross elasticity σ.

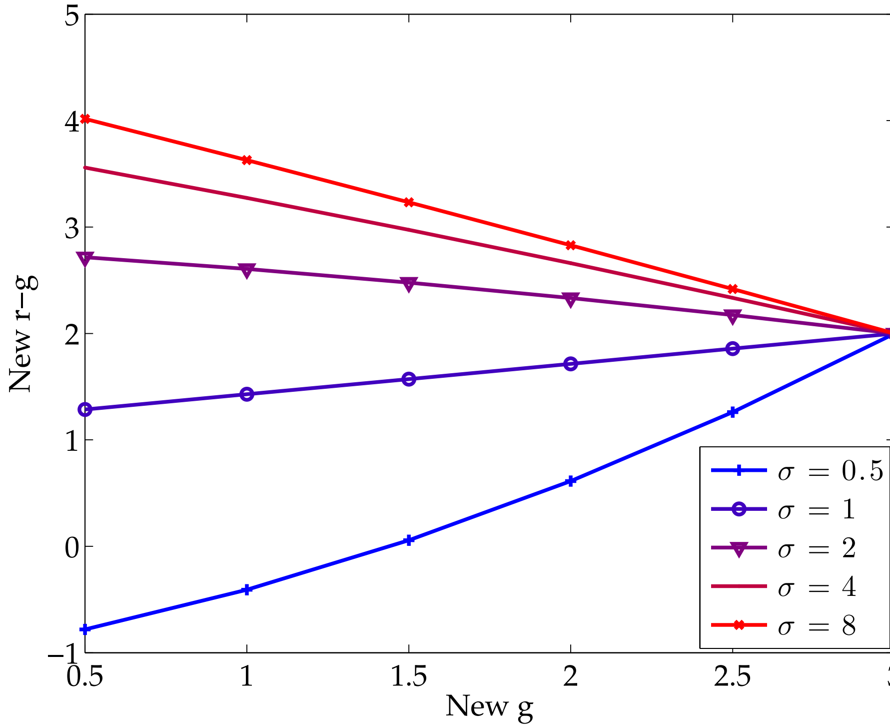


forward-looking agents, but for simplicity I will modify the assumption on savings in a different direction, along the lines of the classical Solow growth model. In this environment, it is *gross* rather than *net* savings that are exogenous, and the long-run ratio of capital to gross income is $K/Y^{gross} = s^{gross}/(g+\delta)$. This weakens the influence of g on the capital/income ratio, and in particular prevents the ratio from rising unboundedly as g approaches 0, as $K/Y^{net} = s/g$ would seem to imply.[9] Effectively, this new assumption means that the net savings rate will fall as the capital stock grows. (See Krusell and Smith (2014) for a much more detailed discussion of the basic Solow model and other models of aggregate saving in the context of Piketty (2014).)

Figure 3 recomputes the results for $r - g$ from Figure 2 using this alternative characterization of savings. Under this alternative assumption, it is possible to obtain much larger increases in $r - g$ for $\sigma^{gross} \geq 2$. As expected, the weaker response of the capital/income ratio to a decline in g lessens the fall in r, and makes it possible in some cases for $r - g$ to expand. But for $\sigma^{gross} = 1$, $r - g$ still declines, and for $\sigma^{gross} = 0.5$ it falls substantially. It still does not appear possible for $r - g$ to increase with any elasticity in the consensus

[9] One subtlety is that when the net savings rate s is exogenous, $g \to 0$ is not even possible under Piketty (2014)'s approach of assuming a constant net elasticity of substitution, because under this assumption capital accumulation alone will always endogenously produce a nonzero rate of growth as the other, exogenous sources of growth tend to zero. Hence in this case, K/Y^{net} stays bounded. This is a notable contrast to the growth literature's traditional dismissal of endogenous growth through capital accumulation alone, and arises because the assumption of a constant net elasticity effectively places a floor on the gross returns from capital, never allowing R to fall below δ regardless of how much capital is accumulated.

Figure 3: Asymptotic $r - g$ following change in g, by gross elasticity σ, given alternative hypothesis on savings.



range of estimates in Table 1.

Of course, moving even further away from Piketty (2014)'s assumption of a constant net savings rate s, it is possible to obtain an increase in $r - g$. One extreme possibility is to assume that s is proportional to g, so that $K/Y^{net} = s/g$ is constant. In this case, $r - g$ mechanically increases as g declines. But then the net capital income share rK/Y^{net} is unchanged as well, which is inconsistent with another critical theme in Piketty (2014)—namely, the rising importance of capital.

This reflects a tension between Piketty (2014)'s two key projections, a rising net capital share rK/Y^{net} and a rising gap $r - g$. Given realistic elasticities, it is impossible to obtain the latter unless we drop the assumption that $K/Y^{net} \propto g^{-1}$ and instead assume that net saving reacts strongly to growth, so that a decline in g causes little increase in K/Y^{net}. But with little increase in K/Y^{net}, there is not much scope for rising capital income, even ignoring the issues discussed in Section 2.3.[10]

[10] This relates to another theme in Piketty (2014), which is the observation that in the distant past, the return r on capital was close to today's levels, even though g was much lower—and that if this historical pattern reasserts itself, we will see an increase in $r - g$ as g declines going forward and r remains stable.

But stable r in the past says little about the elasticity of substitution σ, since the stock of reproducible capital was not very high—most wealth took the form of land for agriculture. The question "how much does accumulation of reproducible capital decrease the net return on this capital?" was not really tested by this experience. If anything, this historical experience suggests that $K/Y = s/g$ with exogenous s is a bad assumption, because K/Y was not nearly as high as near-zero g would suggest. If a decline in

3 Time series on income and capital

3.1 Inferring elasticity from time series data

Under certain conditions, it is possible to infer the elasticity of substitution by looking at the behavior of the share of capital income and the capital/income ratio over time. For instance, returning to (2), we see that the elasticity of the capital share of income with respect to the capital/income ratio is given by

$$\frac{d(\log(F_K K/F))}{d(\log(K/F))} = 1 - \frac{1}{\sigma} \tag{2}$$

If, at any given time, we see both the capital share and the capital/income ratio increasing simultaneously, (2) implies that the local elasticity σ must be greater than 1. (Inversely, if they move in different directions, (2) implies $\sigma < 1$.) Looking at movement over a longer period of time, we can make similar inferences about the relevant average[11] of σ—or, if we assume a constant elasticity, about the underlying parameter σ. Since (2) was derived without specifying whether F is a gross or net production function, these inferences hold for both gross and net elasticities.

Piketty and Zucman (2013) emphasize this as a way of identifying the net elasticity σ, noting that both net capital shares and the capital-income ratio rose over the 1970–2010 period, and that this is roughly consistent with a value of $\sigma = 1.5$. Piketty (2014) cites this observation as its main empirical support for a net elasticity greater than 1.

This observation from Piketty and Zucman (2013) is visible in Figures 4 and 5, where we can see that from 1970 to 2010, on average there have been increases in both the domestic capital to income ratio—with capital valued at market prices—and the net capital income share.[12] The capital/income ratio increased in all countries; the capital share declined in Japan, was roughly neutral in Italy and France, and increased in all other countries. Taking (2) at face value, this generally appears to support a finding of $\sigma > 1$.

3.2 Constant capital prices: a problematic assumption

One important caveat to this analysis is that (2) refers to the *quantity* of capital K, while Piketty and Zucman (2013) and Figure 4 look at the *market value* of capital. If the real price of capital (in terms of output) is constant—an assumption imposed in a model with only

g is not associated with much of an increase in K/Y, then perhaps r will not fall—but only at the cost of invalidating Piketty (2014)'s other projection, a rising capital share. (Regardless, the nature of capital has changed enough in the last several centuries that extrapolation from the distant past is probably an unreliable strategy.)

[11] This average is not a simple arithmetic average. It is the harmonic mean of σ, evaluated along the path of $\log(K/F)$.

[12] I use *domestic* capital and capital income rather than *national*, which is emphasized more in Piketty and Zucman (2013), because we are trying to identify the elasticity of substitution for domestic production. Like Piketty and Zucman (2013), I emphasize "factor-price" income, which excludes taxes on production—we are looking at factor shares and thus want to exclude income that is not paid to any factor of production. All income is stated in net terms.

one type of good—this makes no difference. If there are large changes in the real price of capital, however, the quantity and market value may move in very different directions.

In fact, it turns out that real price changes can induce systematic comovement between the the capital/income ratio and the capital share of income, regardless of the elasticity σ—making inferences about σ from this comovement quite problematic. Capital-augmenting technological change implies similar issues. I show this below with two simple partial equilibrium examples.

Example 1: partial equilibrium with accumulable capital. Suppose that the two factors of production are reproducible capital K and labor L. Assume that the gross production function F is Constant Elasticity of Substitution[13] (CES) with elasticity σ, and that there can be both neutral and factor-augmenting technological change:

$$Y = AF(A_K K, A_L L)$$

Let the real price of capital be P_K, and let the expected rate of inflation be $\pi_K = \dot{P}_K / P_K$. Then taking the net return r as given, the gross capital share $AF_K A_K K$ of income is proportional to

$$P_K^{1-\sigma}(AA_K)^{\sigma-1}(r+\delta-\pi_K)^{1-\sigma} \tag{5}$$

while the ratio of market-value capital to income $P_K K/Y$ is proportional to

$$P_K^{1-\sigma}(AA_K)^{\sigma-1}(r+\delta-\pi_K)^{-\sigma} \tag{6}$$

If the variation in K/Y is being driven by variation in $(r+\delta-\pi_K)^{1-\sigma}$, (5) and (6) will move together if and only if $\sigma > 1$. If the variation in K/Y in induced by variation in prices P_K or technology A_K, however, then they will move in the same direction *regardless* of the elasticity. The mechanism is straightforward: if P_K increases by 10%, then to justify the higher cost of capital goods the marginal product of capital must also increase by 10%. Thus the market value of capital and income from capital move together. In this setting, comovement contains no information about the elasticity.

Here I looked at the gross rather than net elasticity, because it is not possible to define a "net production function" that is independent of the relative price of capital. This is not an issue for the next exercise, which looks at non-depreciating land.

Example 2: partial equilibrium with land. Now suppose that the two factors of production are land T and labor L. Again, assume that the production function F—which is now net, since land does not depreciate—is CES with elasticity σ and admits both neutral and factor-augmenting technological change:

$$Y = AF(A_T T, A_L L)$$

Suppose that the growth rate in total output is constant at g, that T is fixed, and that the growth rate in AA_T is constant at g_T. The rate of growth in the marginal product of T is then $(1-\sigma^{-1})g_T + \sigma^{-1}g$.

[13] Concretely, suppose that F has the form $F(x_1, x_2) = \left(x_1^{\frac{\sigma-1}{\sigma}} + x_2^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$.

Land's share of output is

$$AF_T \frac{A_T T}{Y} \tag{7}$$

Meanwhile, the price of land reflects its capitalized future marginal product, and is thus

$$P_T = \frac{AA_T F_T}{r - (1 - \sigma^{-1})g_T - \sigma^{-1} g}$$

so that the ratio of land at market value to output is

$$\frac{1}{r - (1 - \sigma^{-1})g_T - \sigma^{-1} g} AF_T \frac{A_T T}{Y} \tag{8}$$

Thus the ratio of the market value of land to output (8) is equal to the land share of output (7) times the capitalization factor $1/(r - (1 - \sigma^{-1})g_T - \sigma^{-1} g)$, and they will naturally move together. In fact, in this case, there is no apparent reason to infer anything about σ from the comovement of (7) and (8).[14]

3.3 Empirical importance of capital price changes.

Since changes in the real price of capital make inferences about the elasticity of substitution in Piketty and Zucman (2013) and Piketty (2014) problematic, it is important to know the extent to which such changes have occurred over the 1970–2010 period.

Decomposition of market value wealth over income. Piketty and Zucman (2013) perform a decomposition of annual changes in national wealth, distinguishing between the contribution from net savings and the contribution from real capital gains. The percent contribution q from real capital gains is obtained as a residual, by writing

$$1 + q = \frac{1 + g_w}{1 + (S/W)} \tag{9}$$

where g_w is the actual rate of growth from wealth and S/W, where S is net national savings and W is national wealth, is the rate of growth that would be expected from net savings alone without any real capital gains.

Figure 6 plots the ratio of national wealth to income from 1970 to 2010, removing the contribution from capital gains.[15] The impact is sizable: some countries now show a

[14] Using observable changes in r, g_T, and g, it might be possible to learn about σ, but comovement alone carries no meaning.

[15] This is national wealth rather than domestic capital because Piketty and Zucman (2013) perform the decomposition for national wealth, and I want to keep close to their results. Beyond the fact that domestic capital is a more relevant concept for studying the domestic production function, it is somewhat harder to interpret this decomposition for national wealth because it includes net nominal claims on the rest of the world, and the decrease in the real value of those claims due to inflation contributes negatively to the capital gains component of the decomposition. Fortunately, net nominal claims on the rest of the world are small enough on average (positive for some countries and negative for others) that this does not cause much systematic effect. More generally, the distinction between national and domestic does not seem to make much difference to the results on average, although there are important differences for individual countries. The net nominal claims issue is much more serious if we do the decomposition for private rather than national wealth, since private wealth includes large net positive nominal claims on government.

secular decline in the wealth/income ratio rather than an increase. Table 2 reveals that across the eight countries in the sample, the average change in the wealth/income ratio from 1970 to 2010 was +223%. This turns into only +35%, however, when capital gains are removed. According to this decomposition, $(223 - 35)/223 \approx 84\%$ of the apparent increase in the wealth/income ratio is due to capital gains.

Issues with the decomposition, and the book value alternative. Piketty and Zucman (2013) argue that in (9), using national savings S as ordinarily measured in the national accounts will deliver biased results, because at the time of data collection most national accounts did not include R&D. Since W here is national wealth measured at market value—using market equity, rather than book equity, for corporations—it presumably includes the value of accumulated R&D, meaning that S/W will be biased downward. They suggest an alternative decomposition, with S augmented by a guess for the contribution of R&D to net savings. In this decomposition, real capital gains are lower and account for a smaller (though still significant) share of total capital accumulation.

Another way to account for this bias is to perform the decomposition (9) for *book value* wealth, as measured in national accounts, rather than *market value* wealth. This approach has the advantage of consistency between the savings and wealth series: if the national accounts exclude R&D from savings S, they will also exclude R&D from book value wealth W^{book}.[16] Figure 7 plots the ratio of national wealth at book value to national income from 1970 to 2010, removing the contribution from capital gains. Surprisingly, on average the countries in the sample now show a decline in the ratio, and in some cases (Australia, UK) this decline is dramatic. Table 3 shows that on average, including capital gains the change in national wealth at book value to national income was +198%. Excluding capital gains, however, it is −69%. In other words, capital gains now account for $(198 - (-69))\,/198 \approx 135\%$ of the apparent increase in the wealth/income ratio.

Appendix A.1 discusses the distinction between market and book value at greater length, and the possible causes of this disparity. It concludes that book value is probably the most robust way to measure wealth in this context, as it is more compatible with the data on savings.

General importance of capital prices. Data technicalities aside, Figures 6 and 7 and Tables 2 and 3 strongly suggest that once real price changes are removed, there has not been any robust increase in the capital/income ratio. At the very least, real price changes account for a large share of the overall increase. Given the analysis in Section 3.2, this makes the attempt at inferring σ in Piketty and Zucman (2013) very difficult to sustain.

The importance of capital gains is not surprising in light of the fact that land accounts for a large share of total capital value. As land becomes scarcer relative to output, its relative price is likely to increase. This brings us to the next topic: the decisiveness of housing in trends of capital and capital income growth.

[16] This notion of book value still includes price changes for nonfinancial assets. The only difference between market value and book value national wealth in the Piketty and Zucman (2013) dataset is that market value wealth uses market equity value rather than book equity value for the corporate sector.

3.4 Empirical importance of housing

The analysis in Section 3.2 shows that when capital takes the form of land, there is no reason to infer $\sigma > 1$ from comovement between net capital income and the market value of capital. This makes it crucial to examine the empirical role of assets with a large land component—most notably, housing.

Another reason to be interested in housing is its distinctiveness as a factor of production. The dominant share of housing services is provided directly by houses themselves—although there are other sources of value added (maintenance, insurance, administration of rental units, etc.), these are relatively minor. In this respect, housing is unlike most other goods and services, which are produced with input from many different types of labor and capital in concert. Given the distinctiveness of housing, it can be clarifying to break the aggregate production function into two parts, writing it as

$$G(F(K, L), H) \tag{10}$$

Here, the function $F(K, L)$ combines non-housing capital K and labor L into non-housing goods and services. Then G, reflecting consumer preferences, combines non-housing output $F(K, L)$ and housing output H into a final consumption aggregate. In principle, we can ask two questions about (10): first, the elasticity of substitution between K and L in F; and second, the elasticity of substitution between non-housing output $F(K, L)$ and housing H in G. The implied *aggregate* elasticity between capital and labor will depend on both.[17] Especially if they turn out to be very different, distinguishing between these elasticities will lead to a clearer picture of the growth of capital, as well as the associated policy implications.

Importance of housing in the capital/income ratio. Using data from Piketty and Zucman (2013), Figures 8 and 9 break the domestic capital/income ratio from 1970 to 2010 into two components: housing capital and all other forms of capital. Figure 8 shows an increase in the housing capital/income ratio of over 100 percentage points for all countries in the sample except the US, which had a 43pp increase.

By contrast, the other capital/income ratio in Figure 9 only increased by over 100pp in Japan, and actually decreased in Canada and Germany. Across all eight countries in the sample, the average increase in housing capital/income was 186pp, while the average increase in other capital/income was only 44pp—making housing responsible for roughly 80% of the overall increase.

Important of housing in the capital income/income ratio. Similarly, Figures 10 and 11 break the net domestic capital income/income ratio into two components: net capital income from housing and net capital income from all other sources. Since the available time series for income in most countries extends back to 1960, Figures 10 and 11 also add the 1960s to provide a longer-term perspective.

[17] Section 4.2 discusses theoretically for a production function of the form (10) how these elasticities are aggregated.

The contrast here is even more striking: there has been a large long-term increase in the share of net income from housing for every country in the sample except Germany, for which the data only extends back to 1990. Meanwhile, the non-housing capital share shows no clear trend, with some countries experiencing increases and other decreases. For the sample as a whole, the average increase in the housing share from 1970 to 2010 is 3.4pp, while the non-housing capital share has shown an average *decrease* of 1.9pp. Net housing income thus accounts for over 100% of the increase in the net capital share in this period.

To clarify, Table 4 decomposes the net non-housing capital income shares further, into corporate and proprietor capital income. On average, there has been a substantial long-term increase in the net housing share and a substantial long-term decrease in net proprietor capital share. The share of net corporate capital is roughly unchanged since the 50s and 60s, but it has recovered following a dip in the 70s. Appendix A.2 discusses the possibly surprising absence of any long-term increase in the net corporate capital income share in more detail, and how this can be reconciled with the literature.[18]

Interpreting the role of housing. Together, Figures 9 and 11 show that the fact emphasized by Piketty and Zucman (2013) as justification for a net elasticity greater than 1—the simultaneous long-term rise in the capital/income ratio and the net capital share of income—vanishes once we remove housing. How should we interpret this? Although the primacy of housing is certainly an important story, it may seem arbitrary to ignore housing altogether when making inferences about the aggregate elasticity. After all, housing is a very important part of the capital stock—perhaps the aggregate elasticity is high *because of* housing?

There are a few reasons, however, to be skeptical that housing's enormous contribution to the data tells us very much about the aggregate elasticity. First, as observed in Section 3.2, Piketty and Zucman (2013)'s strategy for inferring the net elasticity is precarious when real capital prices vary, and fails entirely when capital takes the form of land. Since housing is problematic on both counts—its real price varies substantially, and it has a large land component—aggregate results driven entirely by housing are not very convincing.

Futhermore, if housing singlehandedly pushes the aggregate net elasticity above 1, consumers' elasticity of substitution in (10) between housing and other forms of consumption must be quite high.[19] But empirical work on the price elasticity of demand for housing services suggests relatively low values—for instance, Ermisch, Findlay and Gibb (1996) finds values between 0.5 and 0.8 in a review of the literature, and itself provides an estimate of 0.4. Figure 15 displays the real price of housing services[20] against

[18] Like all cross-country time series in this note, these shares are taken directly from the Piketty and Zucman (2013) dataset. In this dataset, proprietors' capital income is imputed as a fraction of total proprietors' income (which also includes labor). Corporate capital income is defined to include *all* capital income originating in the corporate sector—note the contrast with corporate profits—regardless of whether it is paid out to debt or equity. For our purposes, this is the right notion.

[19] Equation (11) in Section 4.2, with K^2 interpreted as housing, shows formally how the elasticity η between housing and other consumption feeds in to an implied aggregate elasticity.

[20] This is the ratio of the price index for housing and the price index for all personal consumption expen-

housing's share of personal consumption expenditures in the United States, showing that as the cost of housing has risen over the last several decades, its share of consumption has also risen slightly. And then there is a well-known tendency for consumers to spend a larger share of their budgets on housing in areas where housing is expensive.[21] All this indicates a *gross* elasticity between housing and other consumption of less than 1. It is unlikely, therefore, that the data emphasized by Piketty and Zucman (2013) reflects a genuinely high elasticity driven by housing.

Indeed, the most consistent overall account of trends in housing invokes an elasticity that is below—not above—one. As housing has become relatively more expensive, both its aggregate value and its share of household expenditures have risen across the countries in Piketty and Zucman (2013)'s sample. And since the higher cost of housing is mainly due to higher residential land values, rather than elevated construction costs for the structures themselves, it has made a particularly large contribution to *net* capital income. (Land does not depreciate; all its income is net.)

This pattern may well continue, in which case Piketty (2014) will be right about the rise of capital in the twenty-first century. But the mechanism is quite distinct from the one proposed by Piketty (2014) (a better title would be *Housing in the Twenty-First Century*), and it has radically different policy implications. For instance, the literature studying markets with high housing costs finds that these costs are driven in large part by artificial scarcity through land use regulation—see Glaeser, Gyourko and Saks (2005) and Quigley and Raphael (2005). A natural first step to combat the increasing role of housing wealth would be to reexamine these regulations and expand the housing supply.

ditures in the national accounts.

[21] For instance, a comparison of Consumer Expenditure Survey evidence across 18 major metropolitan areas showed that New York had the highest expenditure share on housing. See table 3 at `http://www.bls.gov/ro9/cexsanf.htm`.

4 Technology and the net return to capital

4.1 Technology and the rise of capital

Piketty (2014) emphasizes the role of new technology in creating opportunities for capital investment, suggesting that this makes the elasticity of substitution between capital to labor high in the long run:

> Indeed, the observed historical evolutions suggest that it is always possible—up to a certain point, at least—to find new and useful things to do with capital: for example, new ways of building and equipping houses (think of solar panels on rooftops or digital lighting controls), ever more sophisticated robots and other electronic devices, and medical technologies requiring larger and larger capital investments.

There is no doubt that advanced new technology, often embodied in capital investments, has a transformative effect on the economy. As demonstrated by an extensive literature in labor economics, the automation of routine tasks has "polarized" the labor market in recent years, leading to growth in employment at the top and bottom but contraction in the middle. (See, for instance, Autor, Katz and Kearney (2006).) And as Brynjolfsson and McAfee (2014) forcefully argue, the most profound effects of today's technological revolution may be yet to come.

Composition of the capital stock. Does advanced new technology, by creating novel applications of capital, play an important role in holding back diminishing returns? If the large rise in the capital-income ratio envisioned by Piketty (2014) comes to pass, will high-tech equipment absorb much of the increase? Judging by the current composition of capital, at least, this seems unlikely. As Figure 16 reveals, structures continue to comprise the vast majority of the private capital stock in the US—175% of GDP at current prices. The other components of private capital—equipment, intellectual property, and consumer durables—are much smaller, and within them the share of capital related to automation and the information revolution is smaller still. Within equipment, for instance, "information processing equipment" (computers, communication, medical, etc.) is only 8% of GDP, as compared to 27% of GDP for all other equipment. Software is 4% of GDP, versus 11% for other intellectual property. And technology-intensive consumer durables (computers, TVs, phones, etc.) stand at 3% of GDP, as opposed to 27% for other durables.

Figure 17 shows these technology-related components in more detail, revealing that recent trends are mixed. Although the aggregate value of software has risen consistently relative to nominal GDP, information processing equipment actually peaked in the mid-1980s. The quantity of computing power has risen spectacularly, but it has been overwhelmed by a relative price decline. Altogether, it is clear from Figure 17 that the value of these components (15% of GDP) is unlikely to rival structures (175% of GDP) for the foreseeable future.

Why have these forms of capital had such a profound economic impact, even though their aggregate value remains small compared to the value of houses, apartments, and offices? First, since technology has led to a massive decline in the price of high-tech capital (visible in Figures 17-19), the stagnation in *value* at current prices masks a considerable increase in the real *quantity* of this capital. As discussed in Section 3.2, when the real price of capital is not constant, it is the quantity K^{tech} of capital that matters to the production function, rather than the value $P_K^{tech}K^{tech}$. The vast increase in effective quantity—with more computing power in today's smartphones than 1980's supercomputers—has led to great progress, displacing many traditional products and occupations along the way.

But as Piketty (2014) rightly emphasizes, it is the value of capital, rather than the quantity, that matters when analyzing capital wealth and income. Since high-tech capital has experienced roughly offsetting trends in P_K^{tech} and K^{tech}, its value $P_K^{tech}K^{tech}$—and the accompanying net capital return $rP_K^{tech}K^{tech}$—is still relatively small. For this component of capital to absorb a substantial share of Piketty (2014)'s projected rise in aggregate capital value $P_K K/Y^{net} = s/g$, it must experience a vast expansion.

Second, the rapid rate of depreciation and obsolescence for high-tech capital—according to the Bureau of Economic Analysis, 18% for information processing equipment and 43% for software[22]—-means that the net return is only a small fraction of the gross return. The net capital income share $rP_K^{tech}K^{tech}/Y^{net}$ from these assets is small owing to the low P_K, but with rapid depreciation δ^{tech} the gross capital income share $(r + \delta^{tech})P_K^{tech}K^{tech}/Y$ is much greater. Since the gross share is the visible return on capital (we do not mentally subtract the rate of depreciation when we see robots displacing assembly line workers, nor does it matter to the displaced workers), it is natural to overestimate the net contribution.

Elasticity of capital demand. Even if structures dominate the capital stock today, will other forms of capital surge to prominence in the future as they become more affordable? Figure 18 shows the real[23] price of equipment investment versus the equipment/GDP ratio valued at current equipment prices, on a log scale, since 1970 in the US. Since the latter ratio has stagnated even as the real price of equipment has continued to decline, the (gross) elasticity of demand for equipment appears close to 1. There is little indication that the equipment/GDP ratio will rise going forward, even if the secular decline in price continues.

Figures 19 and 20 provide the corresponding trends for computers and software, respectively. Figure 19 shows a rapid decline in the real price of computers coupled with relatively little increase in the computer/GDP ratio, again indicating an elasticity of demand close to 1. Figure 20, on the other hand, shows that the decline in real price of software has been accompanied by a comparable upward trend in the value of software relative to GDP— possibly indicating a higher elasticity. This is worthy of attention, and hints at an increasingly important role for software as time progresses. But the stock of software will have to rise far above its current level of 3% of GDP before it has much impact on net capital income. Its current contribution, assuming $r = 5\%$, amounts to only $5\% \times 3\% = 0.15\%$ of GDP, or 0.18% of NDP.

[22] The depreciation rates for information processing equipment and software are lines 4 and 78 in Fixed Assets Table 2.4 divided by lines 4 and 78 in Fixed Assets Table 2.1.

[23] (Normalized by the GDP deflator.)

4.2 Heterogenous capital and the aggregate elasticity of substitution

I have argued that Piketty (2014) is wrong to emphasize new technologies, which primarily are embodied in equipment, as the source of a high aggregate elasticity of substitution between capital and labor. At current prices, structures are a vastly larger component of the overall capital stock, and they will determine the economy's capacity to productively employ additional capital.

It is helpful to make this argument more concrete. Suppose that aggregate capital K must be split between two components, $K^1 + K^2 = K$. (For simplicity, I normalize K^1 and K^2 to have a real price of 1 in terms of the output good, eliding the dynamic effects of changing capital prices.) K^1, which we can think of as equipment, is combined with labor L using the constant returns to scale technology G to produce services $G(K^1, L)$. These services are aggregated with K^2, which we can think of as structures, to produce final gross output H.

The implied net production function $F(K, L)$ takes the form

$$\begin{aligned} F(K,L) = \quad & \max_{K^1,K^2} \quad H(G(K^1,L),K^2) - \delta^1 K^1 - \delta^2 K^2 \\ & \text{s.t.} \qquad K^1 + K^2 = K, \end{aligned}$$

optimally allocating K between K^1 and K^2 to maximize net output. What is the elasticity of substitution σ for F, and how does it depend on the environment?

The net elasticity σ turns out to be given by the following expression:

$$\sigma = (1-\kappa^1)(\phi^2 - \alpha^1\phi^1)\eta + \left(\kappa^1(1-\alpha^1) + \alpha^1\right)\phi^1\epsilon \tag{11}$$

Here, η is the (gross) elasticity of substitution for H and ϵ is the (gross) elasticity of substitution for G. $\kappa^1 \equiv K^1/K$ is equipment's share of aggregate capital, and $\alpha^1 = G_K K^1/G$ is the gross capital share of production in G. (See Appendix B.4 for the details behind (11).)

ϕ^1 and ϕ^2 are given by

$$\phi^i = \zeta^i + \gamma(1-\zeta^i) \tag{12}$$

where $\zeta^i = r/(r+\delta^i)$ is the net share of the gross return from capital K^i, and γ is the net capital share of production in F. One can show that $\phi^1\epsilon$ is the net elasticity of substitution for G. Assuming that γ is well below 1 (as in most economies), when the net share $\zeta^1 = r/(r+\delta^1)$ is small the factor ϕ^1 is also small, indicating a large gap between net and gross elasticities.

Although (11) may seem forbidding, it has a straightforward interpretation as a weighted sum of two elasticities: the elasticity of substitution η between services G and structures K^2, and the elasticity of substitution ϵ between K^1 and L for G. These correspond to the two margins along which we can substitute capital and labor.

I will focus on the coefficient $\left(\kappa^1(1-\alpha^1) + \alpha^1\right)\phi^1$ in (11) on the elasticity of substitution ϵ between equipment K^1 and labor L. For the purposes of rough quantitative illustration, consider the parameter choices:

- $\phi^1 = \zeta^1 + \gamma(1-\zeta^1) \approx 50\%$.

  - This corresponds to $\zeta^1 = r/(r+\delta^1) \approx 27\%$ given $r = 5\%$ and $\delta^1 = 13.5\%$ (the average depreciation rate of equipment in the US), and also $\gamma \approx 32\%$ (the net capital share of US net factor-price domestic income).

- $\kappa^1 = 16\%$.

  - This is (equipment stock)/(equipment stock + structures stock) for the US.

- $\alpha^1 = 15\%$.

  - This is a rough guess for the gross capital share of output when all structures are excluded.

Given these parameter values, the coefficient on ϵ is

$$\left(\kappa^1(1-\alpha^1)+\alpha^1\right)\phi^1 \approx .14$$

Hence the gross elasticity of substitution ϵ between equipment and labor contributes relatively little to the net aggregate elasticity of substitution σ in (11). This is for two primary reasons. First, the equipment share κ^1 of the aggregate capital stock is quite low, limiting the role of equipment in absorbing additional capital at the aggregate level. Second, in an echo of Section 2, the net elasticity of substitution $\phi^1\epsilon$ is already much smaller than the gross elasticity ϵ. Since depreciation accounts for a larger share of its costs, equipment is less sensitive than structures to the net cost of capital r, further diminishing its role in the aggregate substitution of capital and labor.

This calculation is little more than a formal confirmation of the intuition from Section 4.1. Structures account for the vast majority of the capital stock, and it is unsurprising that they play a key role in determining the economy's overall elasticity. For now, diminishing returns to capital at the aggregate level are driven by the same unexceptional forces as ever—most of all, by the extent to which building more homes, offices, hospitals and shopping centers brings down their incremental value.

5 Concluding remarks

Compared to the grand scope of Piketty (2014), the objective of this note has been quite narrow: to systematically explore the relevant evidence on diminishing returns to capital. Technical and uninspiring as this question may be, it is an essential step in knowing whether the prediction of rising capital income and inequality through accumulation—a prediction that gives *Capital in the Twenty-First Century* its title—will really come to pass. And given the evidence here that Piketty (2014) understates the role of diminishing returns, some skepticism is certainly in order.

But rejection of this specific mechanism does not constitute rejection of all Piketty (2014)'s themes. Inequality of labor income, for instance, is a very different issue—one that remains valid and important. Capital itself remains an important topic of study. Among large developed economies, the remarkably consistent trend toward rising capital values and income is undeniable. As Sections 3.3 and 3.4 establish, this trend is a story of rising capital prices and the ever greater cost of housing—not the secular accumulation emphasized in *Capital*—but it has distributional consequences all the same. Policymakers would do well to keep this in mind.

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A Selected discussion

A.1 Book vs. market value in Section 3.3 decomposition

In the decomposition in Section 3.3, why does the move from market value to book value, intended to fix the positive bias from unmeasured R&D, actually imply an even larger contribution from capital gains—enough to wipe out the entire measured increase in the book value wealth to income ratio? The reason is simple: for most country $\times$ year observations in the Piketty and Zucman (2013), aggregate Tobin's q is less than 1—the book value of wealth exceeds the market value. And since $W^{book} > W^{market}$, the rate of growth in book value S/W^{book} implied by net savings is less than the corresponding figure S/W^{market} for market value. This increases the implied size of the capital gains component in the decomposition (9).

Of course, there is a broader puzzle here: if R&D and other forms of intangible capital are included in W^{market} but not W^{book}, why is $W^{book} > W^{market}$? There are many possible reasons, and most suggest that it is preferable to evaluate (9) for book rather than market value. One possibility is that investment in the national accounts is overstated, leading W^{book} (which is computed in the national accounts from investment flows using a perpetual inventory method) to be overstated. In this case, however, net saving S will also be overstated, and it is better to use the ratio S/W^{book} (where both numerator and denominator are too high) than S/W^{market} (where only the numerator is off).

Alternatively, if corporate market equity value is lower than properly measured book equity value, it may be that assets are less valuable in corporate hands—due, perhaps, to poor shareholder control. (Piketty and Zucman (2013) discuss this hypothesis, which seems plausible in light of the cross-country pattern in Tobin's q.) In this case, we expect $1 of net corporate saving to contribute less than $1 to market value, and when performed using market value the decomposition (9) should be modified accordingly. Book value remains the more robust choice, reflecting the consistency between measurement of S and measurement of W^{book}.

A.2 Understanding the corporate capital share in Section 3.4

How can the absence of any increase in the net corporate capital income as a share of domestic income here be reconciled with other research showing an increase in the capital share of corporate income—most recently, Karabarbounis and Neiman (2014)? One possibility would be a long-term decline in corporate value added as a share of domestic income, but this turns out not to be the case: as Figure 12 shows, net capital income as a share of net corporate value added shows no clear long-term trend, though it retains the drop in the 70s and subsequent recovery.

Instead, the difference between gross and net concepts appears to be key: as Figure 13 shows, there is some long-term increase in the gross corporate capital share. The widening gap between gross and net is due to a striking upward secular trend in consumption of fixed capital as a share of corporate value added. Since Karabarbounis and Neiman (2014) look at gross rather than net capital income, they see an increase. Furthermore, their sample starts in the mid-70s, when corporate capital income was at a low point relative to previous decades; the influence of this base period is also likely important.

B More detailed derivations

B.1 Figure 1

Here we need to calculate how r changes from r^1 to r^2 as $K/(Y-\delta K)$ changes from $K^1/(Y^1-\delta K^1)$ to $K^2/(Y^2-\delta K^2)$. More precisely, we know the ratio of $K^2/(Y^2-\delta K^2)$ to $K^1/(Y^1-\delta K^1)$, as well as $\delta K^1/Y^1$, δ, and r^1, and we want to calculate r^2.

To do so, first use the identity

$$\frac{(\delta K^2/Y^2)^{-1}-1}{(\delta K^1/Y^1)^{-1}-1} = \left(\frac{K^2/(Y^2-\delta K^2)}{K^1/(Y^1-\delta K^1)}\right)^{-1}$$

to compute $\delta K^2/Y^2$. Then use the identity

$$\frac{K^2/Y^2}{K^1/Y^1} = \frac{K^2/(Y^2-\delta K^2)}{K^1/(Y^1-\delta K^1)} \cdot \frac{1-\delta K^2/Y^2}{1-\delta K^1/Y^1}$$

to compute the change in the ratio of capital to gross income, $(K^2/Y^2)/(K^1/Y^1)$. From this and the gross elasticity σ^{gross}, we know the change in the gross return on capital

$$\frac{r^2+\delta}{r^1+\delta} = \left(\frac{K^2/Y^2}{K^1/Y^1}\right)^{-1/\sigma^{gross}} \tag{13}$$

which we can solve for r^2 using our knowledge of r^1 and δ, giving us the change in the net return on capital r^2/r^1.

Finally, we multiply this change in net return by the change in the capital-net income ratio (which we have known from the beginning) to obtain the object of interest, the change in the net capital share:

$$\frac{r^2}{r^1} \cdot \frac{K^2/(Y^2-\delta K^2)}{K^1/(Y^1-\delta K^1)}$$

B.2 Figures 2 and 3

For Figure 2, we first use Piketty (2014)'s "Second Law of Capitalism" with exogenous s, which implies that the ratio of capital to net income is inversely proportional to g.

$$\frac{K^2/(Y^2-\delta K^2)}{K^1/(Y^1-\delta K^1)} = \left(\frac{g^2}{g^1}\right)^{-1}$$

Then we use the same methods and assumptions above initial values as in Section B.1 above to find the implied asymptotic change in net return from r^1 to r^2 following a change from g^1 to g^2.

3 is similar, except that we use

$$\frac{K^2/Y^2}{K^1/Y^1} = \left(\frac{g^2+\delta^2}{g^1+\delta^1}\right)^{-1}$$

and then repeat the calculations from Section B.1 starting from (13).

B.3 Example 1 in Section 3.2

Using the constant elasticity of substitution form $F(x_1, x_2) = \left(x_1^{\frac{\sigma-1}{\sigma}} + x_2^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$ for F, we can also compute $F_1(x_1, x_2) = \left(\frac{x_1}{F}\right)^{-1/\sigma}$, implying that the marginal product of K is

$$AA_K F_K(A_K K, A_L L) = AA_K \left(\frac{A_K K}{F}\right)^{-1/\sigma}$$

which, when equated with the user cost $P_K(r+\delta-\pi_K)$, gives

$$\begin{aligned} AA_K \left(\frac{A_K K}{F}\right)^{-1/\sigma} &= P_K(r+\delta-\pi_K) \\ \frac{A_K K}{F} &= (AA_K)^{\sigma} P_K^{-\sigma}(r+\delta-\pi_K)^{-\sigma} \\ \frac{K}{Y} &= (AA_K)^{\sigma-1} P_K^{-\sigma}(r+\delta-\pi_K)^{-\sigma} \end{aligned}$$

(where the last line uses $Y = AF$), so that the gross capital share of output is

$$P_K(r+\delta-\pi_K)\frac{K}{Y} = P_K^{1-\sigma}(AA_K)^{\sigma-1}(r+\delta-\pi_K)^{1-\sigma}$$

and the ratio of market-value capital to output $P_K K/Y$ is

$$P_K^{1-\sigma}(AA_K)^{\sigma-1}(r+\delta-\pi_K)^{-\sigma}$$

B.4 Elasticity in Section 4.2

(In the interest of brevity, some intermediate steps in this derivation are passed over.)

We are interested in finding the elasticity σ of the implied net production function

$$\begin{aligned} F(K,L) = \quad & \max_{K^1,K^2} \quad H(G(K^1,L),K^2) - \delta^1 K^1 - \delta^2 K^2 \\ & \text{s.t.} \qquad K^1 + K^2 = K, \end{aligned}$$

where H and G are constant returns to scale. To derive σ transparently, I will totally differentiate in logs, using the convention $\hat{x} = d(\log x)$.

First, observe that both the objective and constraint are homogenous of degree 1 in K and L, implying that the $F(K, L)$ is itself constant returns to scale. Let r and w denote the factor rental rates of capital and labor in terms of the output numeraire, respectively, and let γ be the (net) share of capital in F and $(1-\gamma)$ be the share of labor. Then

$$\gamma\hat{r} + (1-\gamma)\hat{w} = 0 \tag{14}$$

Letting ϵ be the gross elasticity of substitution in G, we have $K^1/L \propto ((r+\delta^1)/w)^{-\epsilon}$, or $\hat{K}^1 - \hat{L} = -\epsilon\left(\widehat{r+\delta^1} - \hat{w}\right)$. Defining $\zeta^i \equiv r/(r+\delta^i)$, we can write $\widehat{r+\delta^i} = \zeta^i\hat{r}$, and combining with (14) we have

$$\hat{K}^1 - \hat{L} = -\epsilon\left(\zeta^1\hat{r} - \hat{w}\right) = -\epsilon\phi^1(\hat{r} - \hat{w}) \tag{15}$$

where we define $\phi^i \equiv \zeta^1 + \gamma(1-\zeta^1)$. Combining with the log-linearized production function $\alpha^1\hat{K}^1 + (1-\alpha^1)\hat{L} = \hat{G}$, where α^1 is the gross capital share of G production, we can obtain

$$\begin{aligned}\hat{K}^1 &= \hat{G} - (1-\alpha^1)\epsilon\phi^1(\hat{r} - \hat{w}) \\ \hat{L}^1 &= \hat{G} + \alpha^1\epsilon\phi^1(\hat{r} - \hat{w})\end{aligned}$$

Now, letting $\kappa^1 \equiv K^1/K$, we can aggregate by writing

$$\hat{K} - \hat{L} = \kappa^1\hat{K}^1 - \hat{L} + (1-\kappa^1)\hat{K}^2 = (\kappa^1 - 1)(\hat{G} - \hat{K}^2) - \left(\kappa^1(1-\alpha^1) + \alpha^1\right)\phi^1\epsilon(\hat{r} - \hat{w}) \tag{16}$$

To find the elasticity, we need to write the entire right side in terms of $\hat{r} - \hat{w}$. H has two inputs: G and K^2 . Let the cost of the G input (again expressed in terms of the numeraire, final output) be p^1, and the cost of the K^2 input be p^2. Then if η is the elasticity of substitution of H, we have

$$\hat{G} - \hat{K}^2 = -\eta(\hat{p}^1 - \hat{p}^2) \tag{17}$$

If α^1 is the gross capital share of G, we have $\hat{p}^1 = \alpha^1\widehat{r+\delta^1} + (1-\alpha^1)\hat{w} = \alpha^1\zeta^1\hat{r} + (1-\alpha^1)\hat{w}$. Similarly, $\hat{p}^2 = \widehat{r+\delta^2} = \zeta^2\hat{r}$. Thus, applying $\zeta^i\hat{r} - \hat{w} = \phi^i(\hat{r} - \hat{w})$, we obtain

$$\begin{aligned}\hat{p}^1 - \hat{p}^2 &= (\alpha^1\zeta^1 - \zeta^2)\hat{r} + (1-\alpha^1)\hat{w} = \alpha^1(\zeta^1\hat{r} - \hat{w}) - (\zeta^2\hat{r} - \hat{w}) \\ &= (\alpha^1\phi^1 - \phi^2)(\hat{r} - \hat{w})\end{aligned} \tag{18}$$

Substituting (18) into (17) and then (16), we have

$$\hat{K} - \hat{L} = (1-\kappa^1)\eta(\alpha^1\phi^1 - \phi^2)(\hat{r} - \hat{w}) - \left(\kappa^1(1-\alpha^1) + \alpha^1\right)\phi^1\epsilon(\hat{r} - \hat{w})$$

implying an overall net elasticity of

$$\sigma = (1-\kappa^1)(\phi^2 - \alpha^1\phi^1) - \left(\kappa^1(1-\alpha^1) + \alpha^1\right)\phi^1\epsilon$$

C Tables and figures

All tables and figures here except Figures 15 through 20 are produced using the Piketty and Zucman (2013) dataset, with data currently available at `http://piketty.pse.ens.fr/en/capitalisback`. Figures 4 through 14 intentionally use the same graphical design (country markers, etc.) as Piketty and Zucman (2013) in order to facilitate easy comparability.

Table 2: National wealth at market value/factor-price national income from 1970 to 2010, both actual and excluding capital gains from 1970 to 2010.

	1970	2010 (actual)	change	2010 (no CG)	change (no CG)	CG rate
USA	445%	467%	+22%	334%	-111%	+0.8%
Japan	386%	681%	+295%	486%	+100%	+0.8%
Germany	354%	469%	+115%	548%	+194%	-0.4%
France	406%	710%	+304%	497%	+91%	+0.9%
UK	364%	605%	+241%	271%	-93%	+2.0%
Italy	286%	720%	+434%	401%	+115%	+1.5%
Canada	331%	471%	+140%	400%	+69%	+0.4%
Australia	433%	665%	+233%	346%	-87%	+1.6%
Average	376%	599%	+223%	410%	+35%	+1.0%

Table 3: National wealth at book value/factor-price national income from 1970 to 2010, both actual and excluding capital gains from 1970 to 2010.

	1970	2010 (actual)	change	2010 (no CG)	change (no CG)	CG rate
USA	471%	482%	+11%	343%	-128%	+0.9%
Japan	499%	879%	+380%	477%	-22%	+1.5%
Germany	486%	640%	+154%	540%	+54%	+0.4%
France	476%	862%	+386%	524%	+48%	+1.3%
UK	519%	570%	+50%	342%	-177%	+1.3%
Canada	433%	576%	+143%	365%	-68%	+1.1%
Australia	542%	805%	+263%	353%	-189%	+2.1%
Average	489%	688%	+198%	421%	-69%	+1.2%

(book value data not available for Italy)

Table 4: Decadal averages for net domestic capital income share, broken into corporate, housing, and proprietor income shares.

		1950s	1960s	1970s	1980s	1990s	2000s
USA	Corporate	13%	14%	12%	12%	12%	12%
	Housing	5%	6%	5%	6%	7%	7%
	Proprietor	6%	5%	4%	4%	4%	5%
Japan	Corporate		24%	21%	21%	18%	17%
	Housing		4%	3%	4%	5%	6%
	Proprietor		5%	4%	2%	2%	1%
Germany	Corporate					16%	20%
	Housing					3%	3%
	Proprietor					5%	6%
France	Corporate	10%	11%	11%	11%	13%	12%
	Housing	3%	4%	5%	6%	8%	9%
	Proprietor	8%	6%	4%	3%	3%	2%
UK	Corporate	20%	18%	13%	16%	17%	17%
	Housing	1%	2%	3%	4%	5%	6%
	Proprietor	3%	3%	2%	2%	3%	3%
Italy	Corporate					20%	20%
	Housing					4%	5%
	Proprietor					9%	8%
Canada	Corporate		17%	18%	20%	17%	22%
	Housing		6%	5%	7%	9%	7%
	Proprietor		2%	2%	1%	1%	1%
Australia	Corporate		16%	14%	16%	18%	20%
	Housing		2%	4%	6%	7%	6%
	Proprietor		5%	3%	3%	2%	2%
50s sample	Corporate	15%	14%	12%	13%	14%	14%
	Housing	3%	4%	4%	5%	7%	7%
	Proprietor	6%	5%	3%	3%	3%	3%
60s sample	Corporate		17%	15%	16%	16%	17%
	Housing		4%	4%	5%	7%	7%
	Proprietor		4%	3%	2%	2%	2%

Figure 4: Domestic capital/factor price domestic income ratios 1970-2010

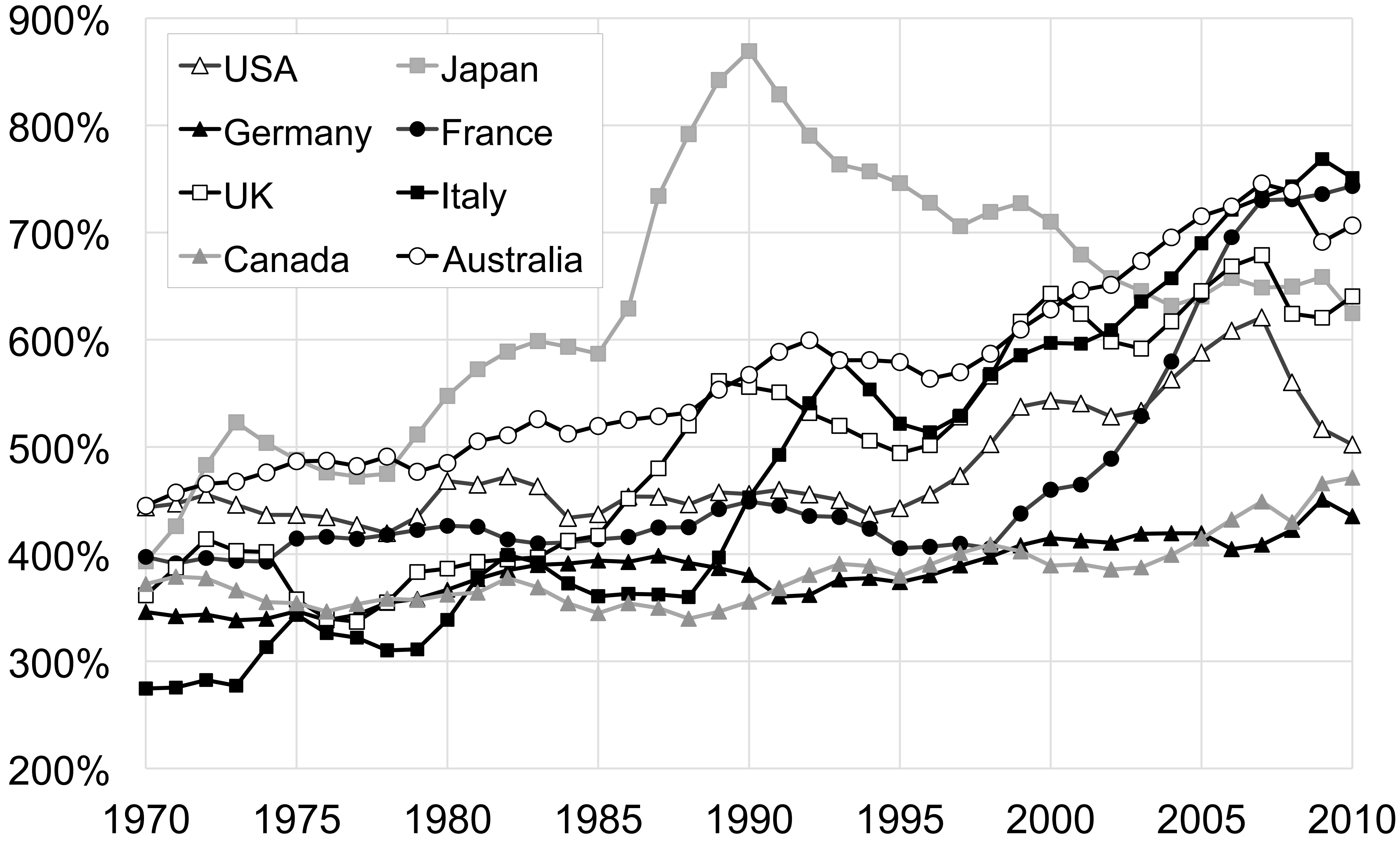


Figure 5: Net domestic capital income/factor price domestic income ratios 1970-2010

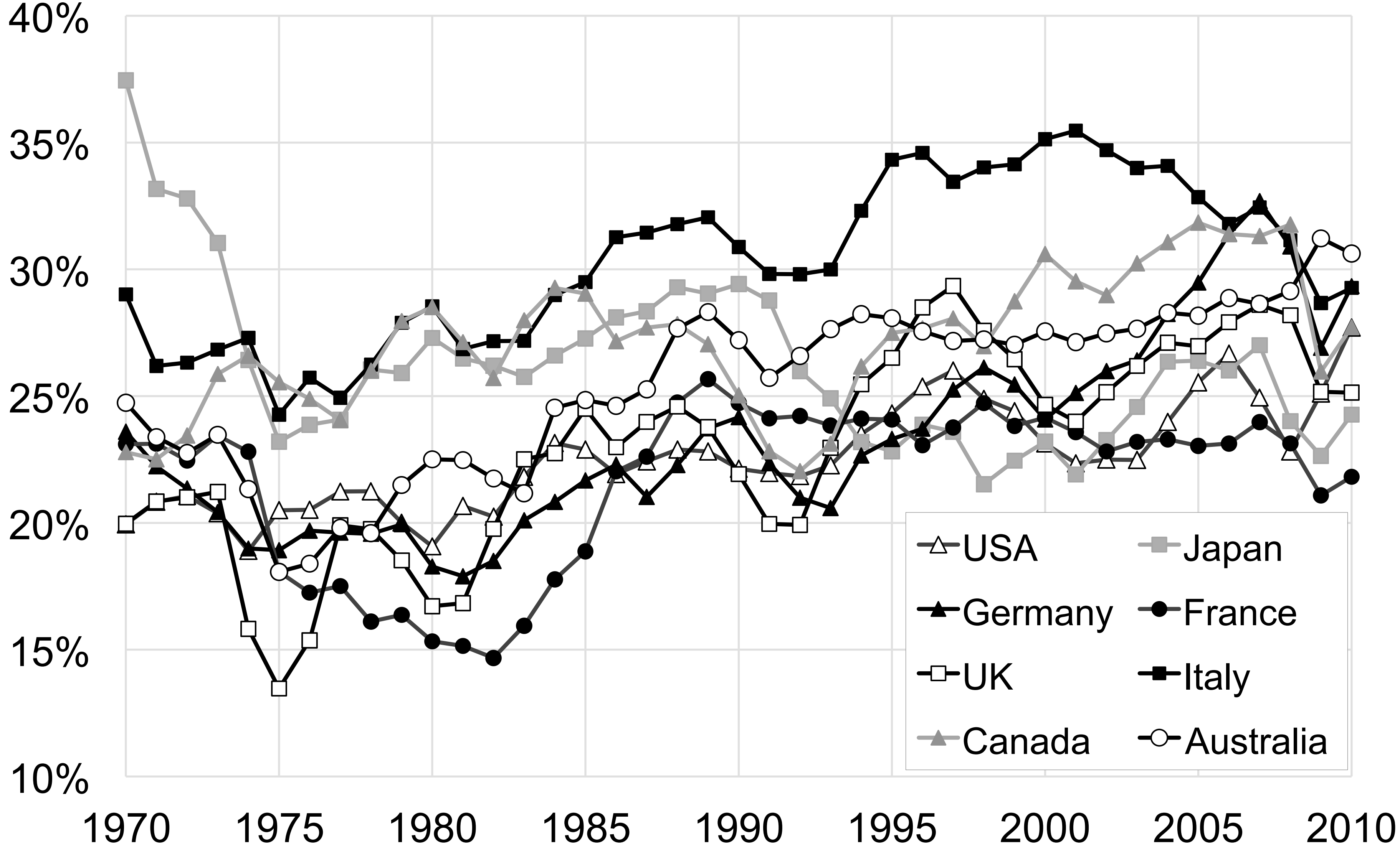


Figure 6: National wealth (market value)/factor price national income ratios 1970-2010, excluding price changes since 1970.

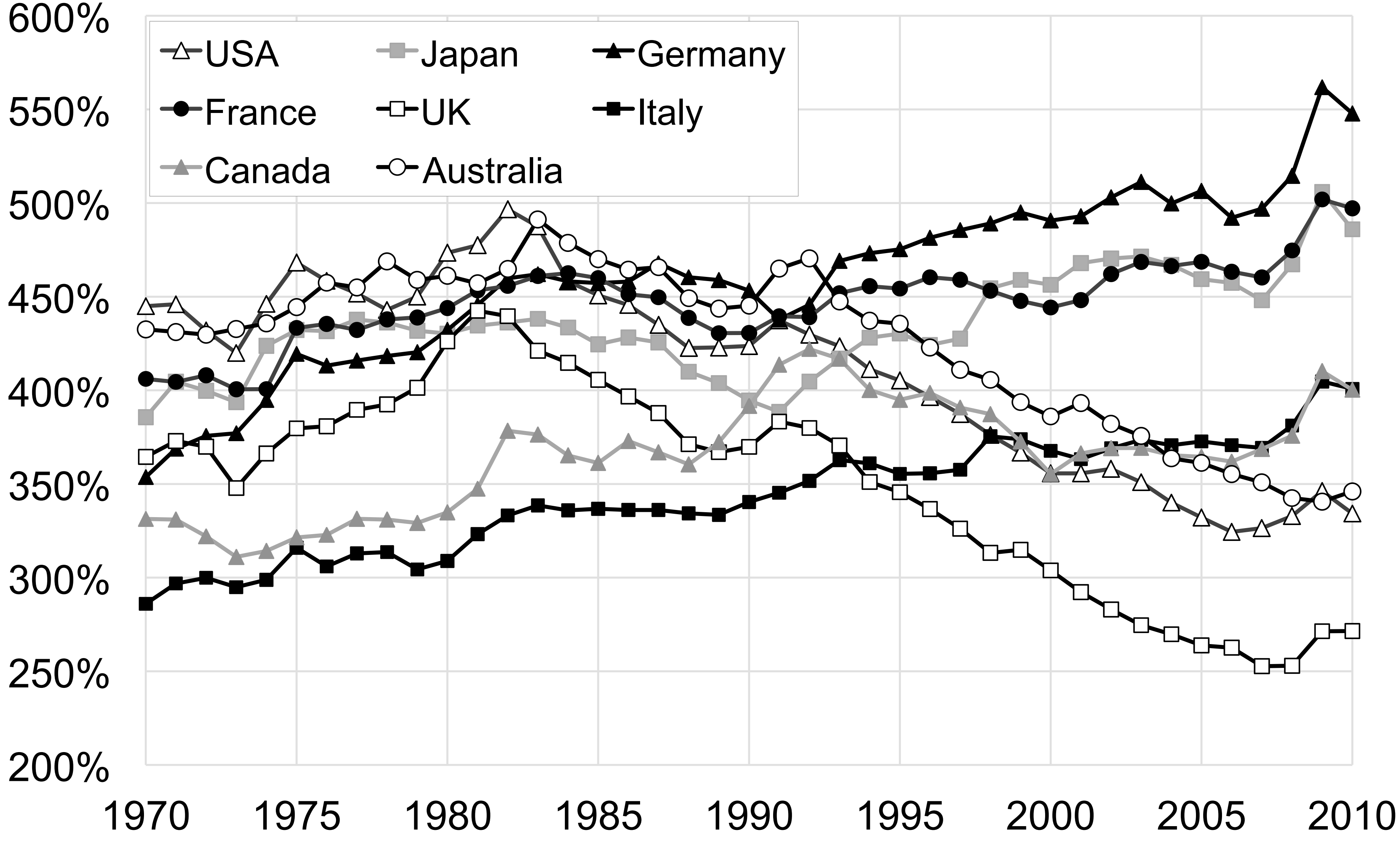


Figure 7: National wealth (book value)/factor price national income ratios 1970-2010, excluding price changes since 1970.

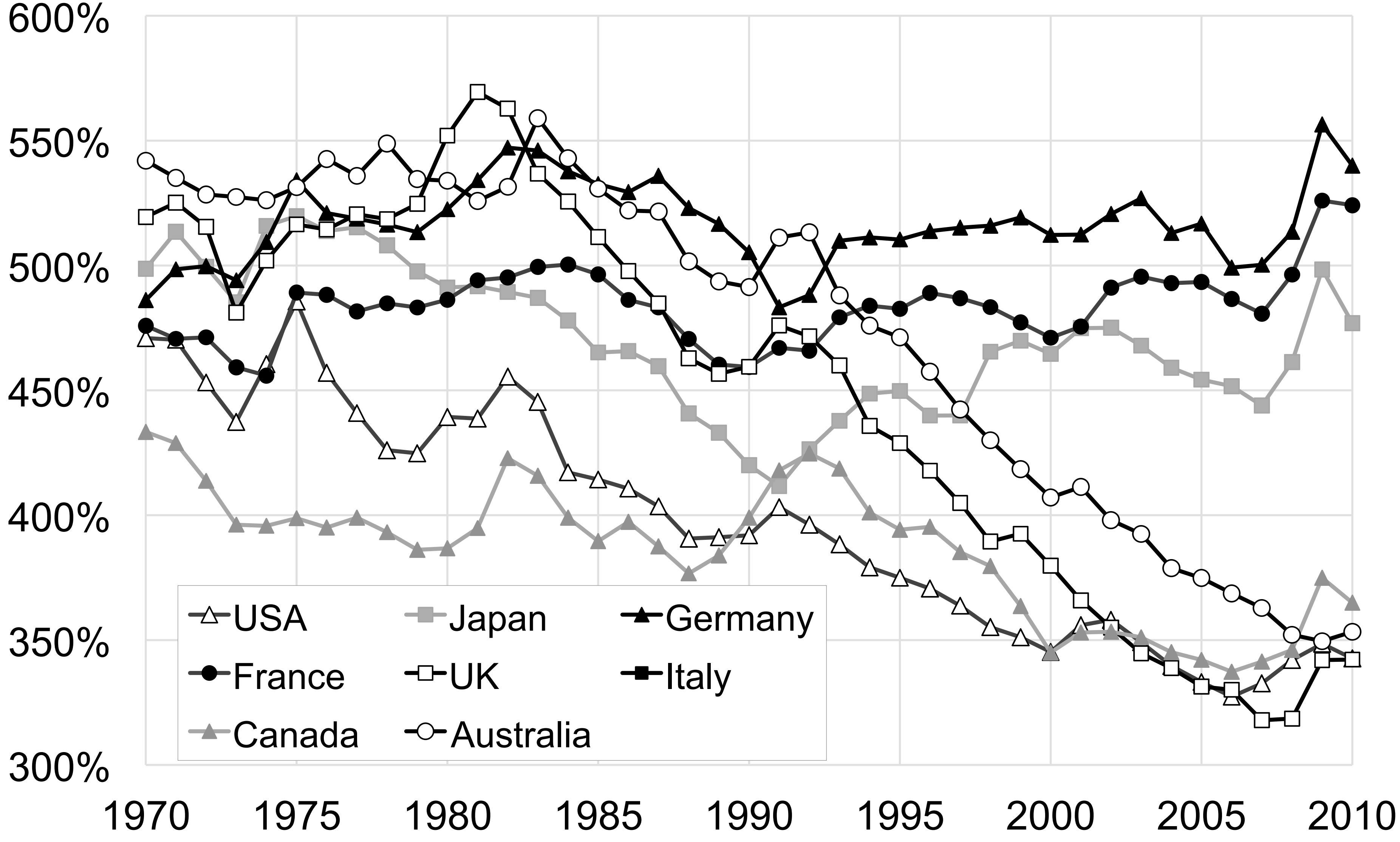


Figure 8: Housing capital/factor price domestic income ratios 1970-2010

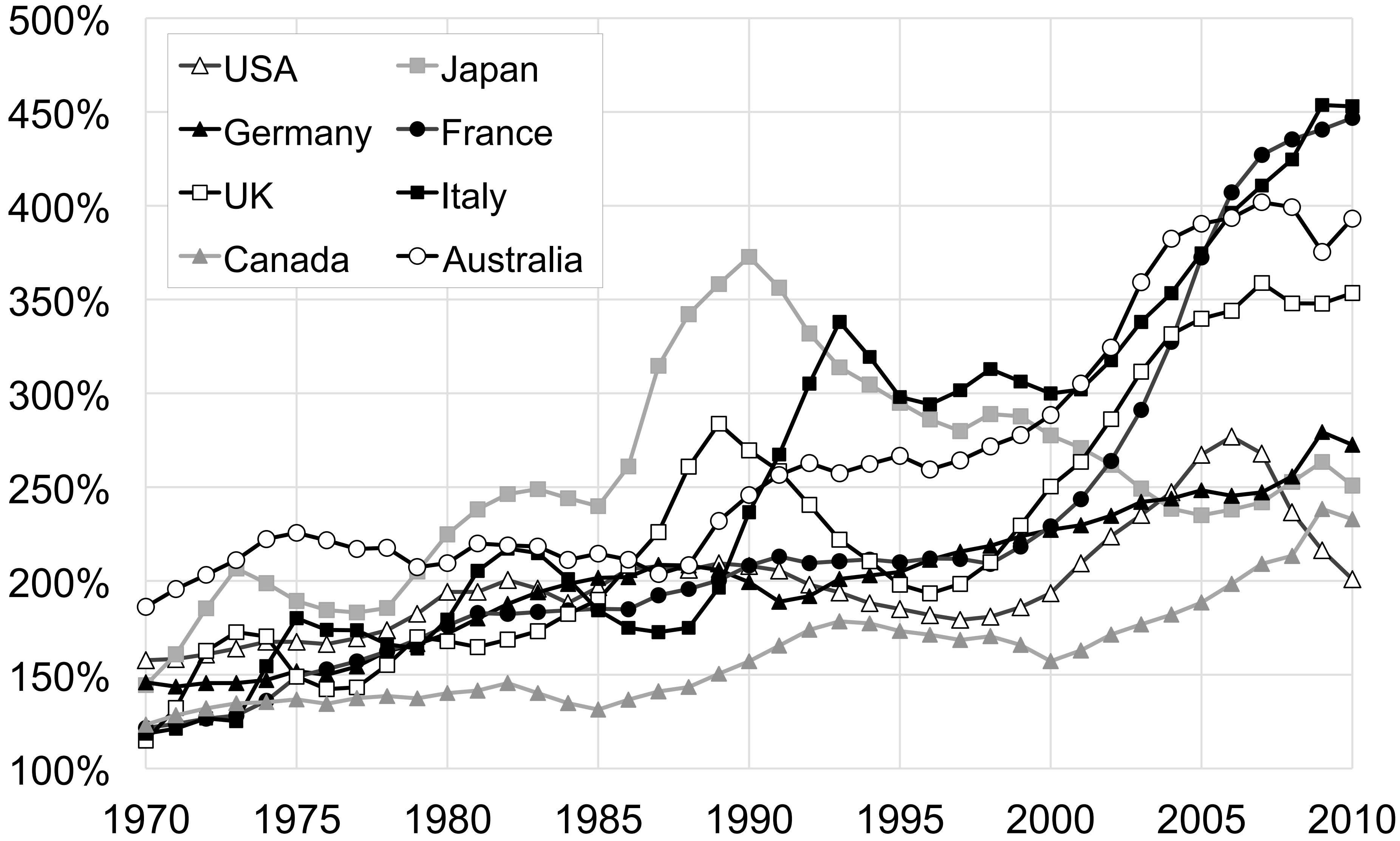


Figure 9: Domestic capital excluding housing/factor price domestic income ratios 1970-2010

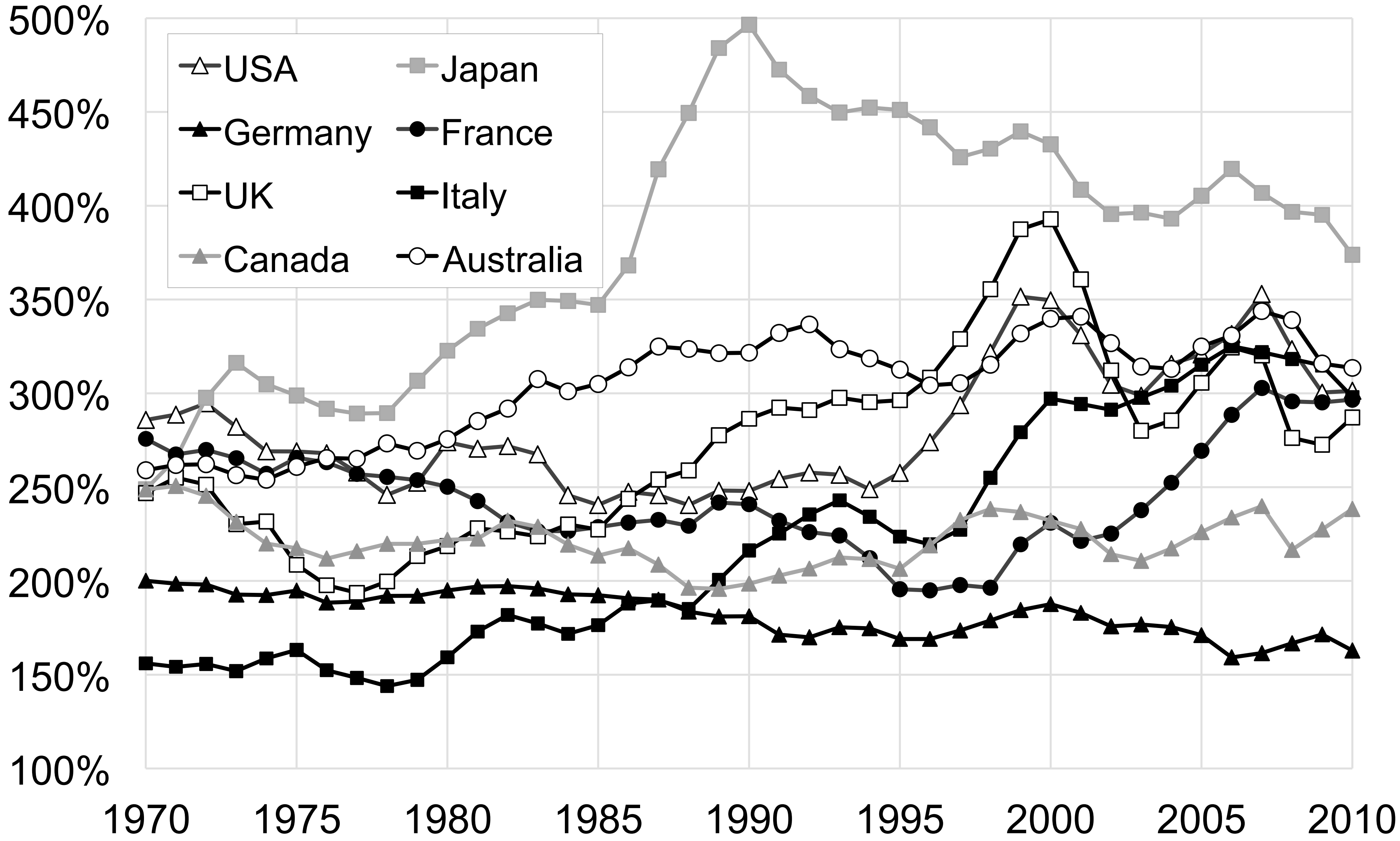


Figure 10: Net housing capital income/factor price domestic income ratios 1960-2010

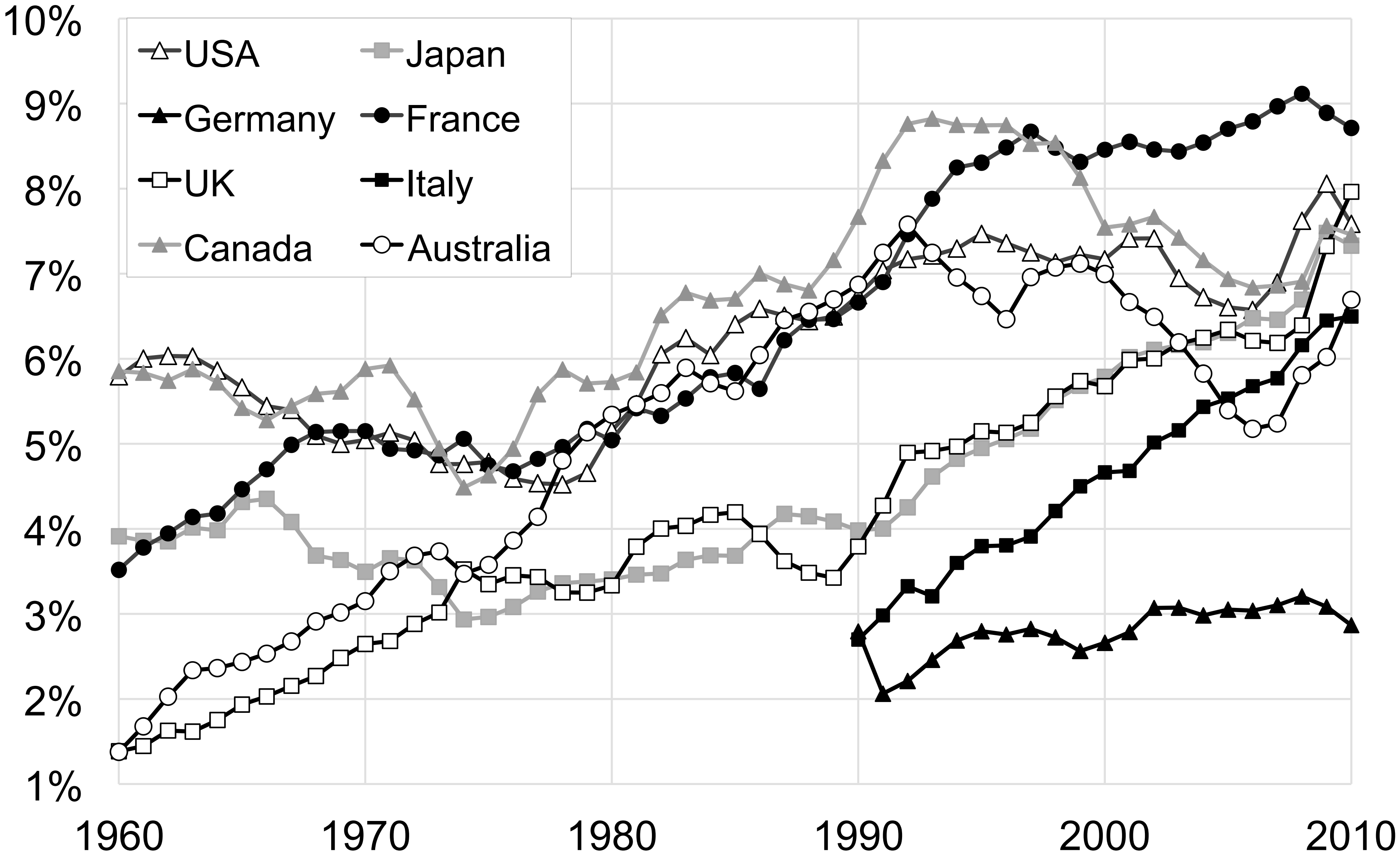


Figure 11: Net domestic capital income excluding housing/ factor price domestic income ratios 1960-2010

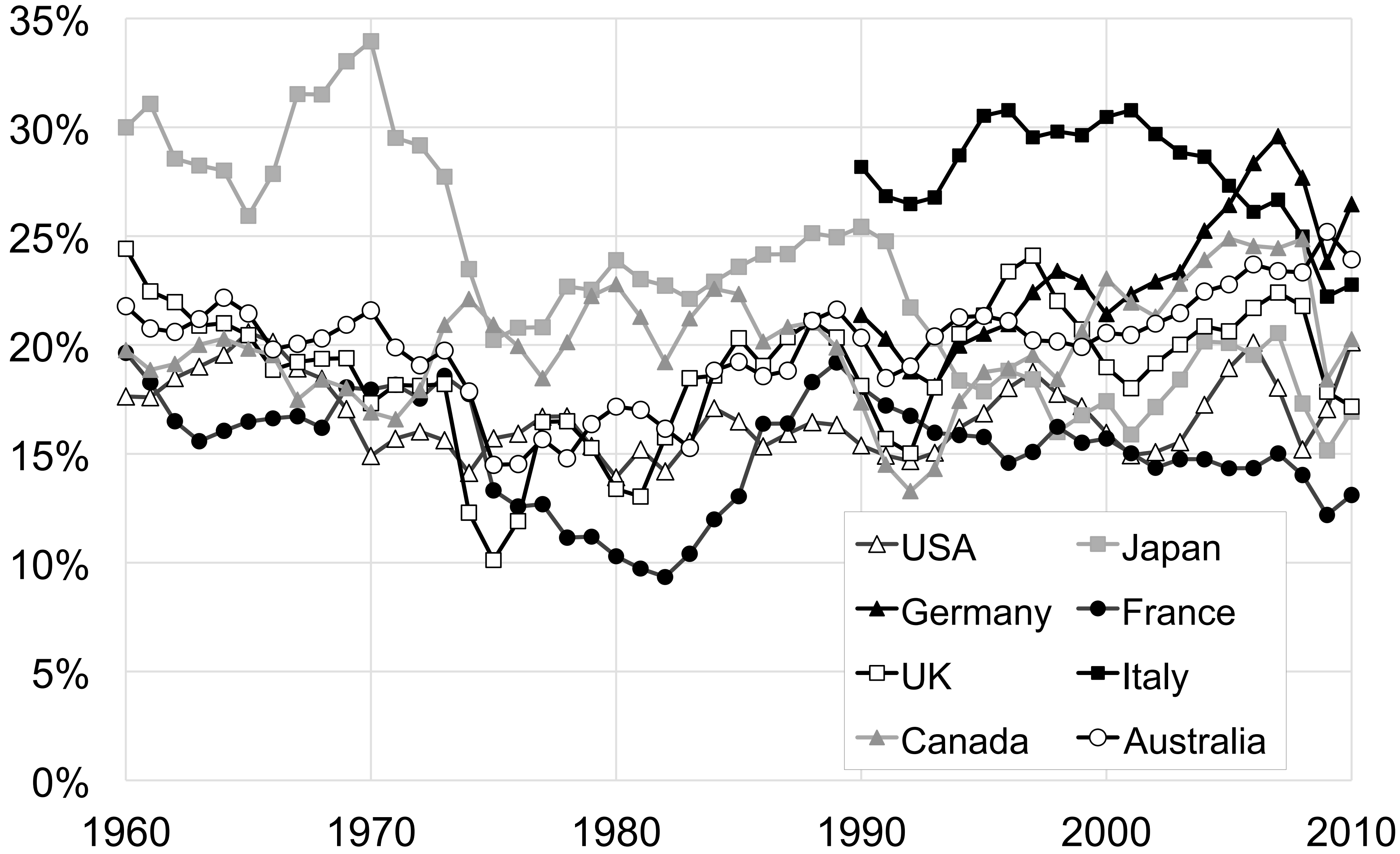


Figure 12: Net capital income share of net corporate value added 1960-2010

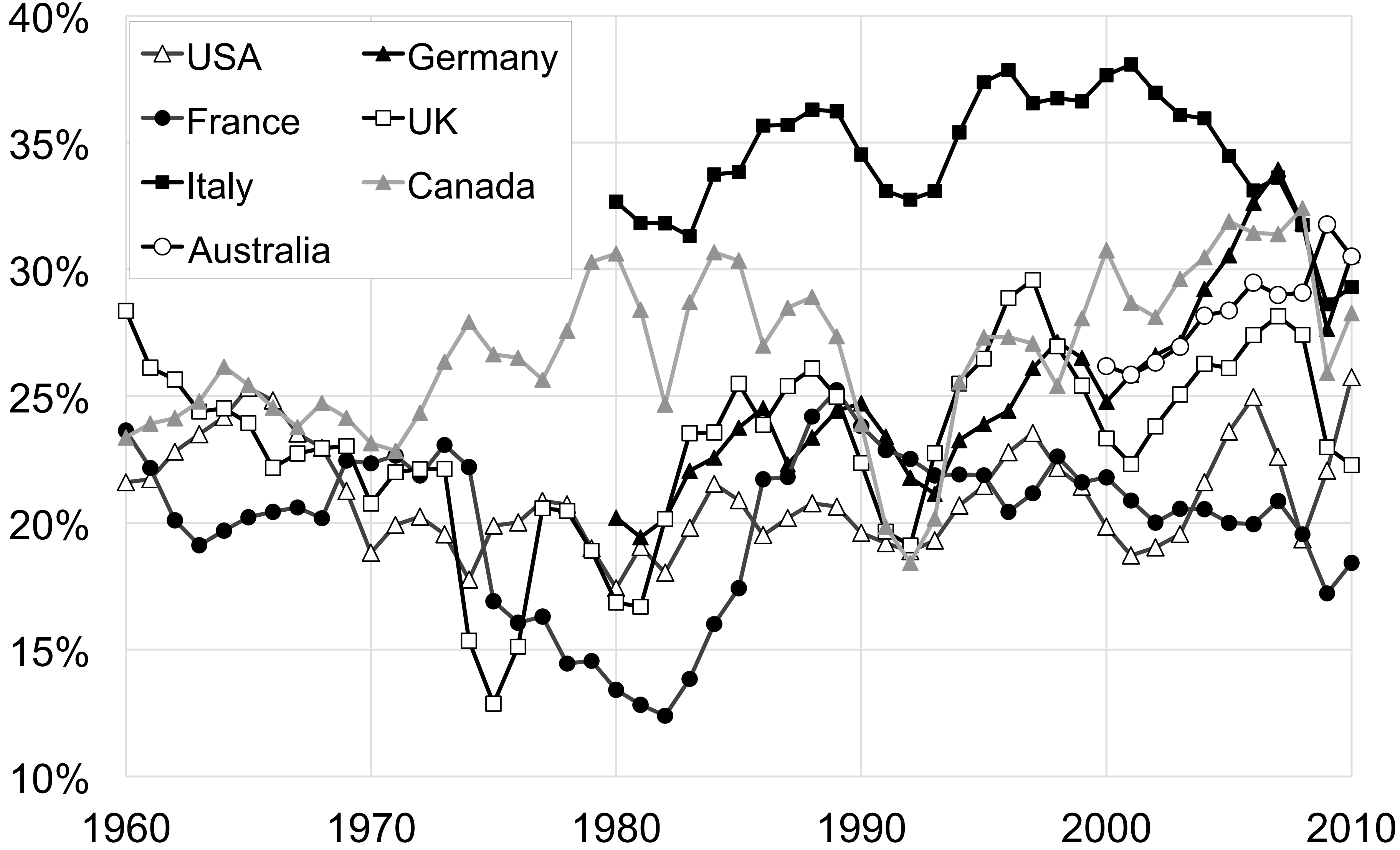


Figure 13: Gross capital income share of gross corporate value added 1960-2010

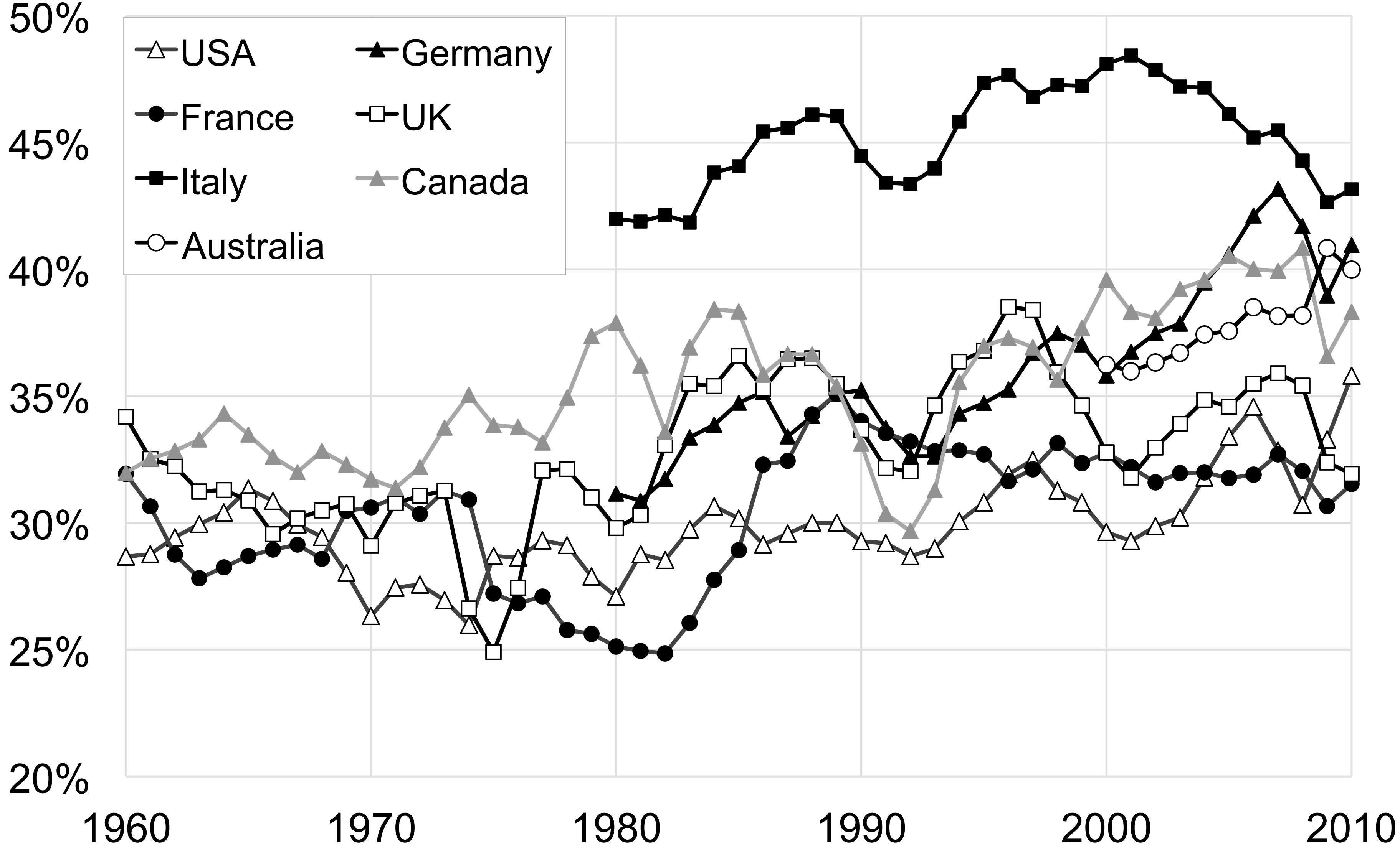


Figure 14: Consumption of fixed capital share of gross corporate value added 1960-2010

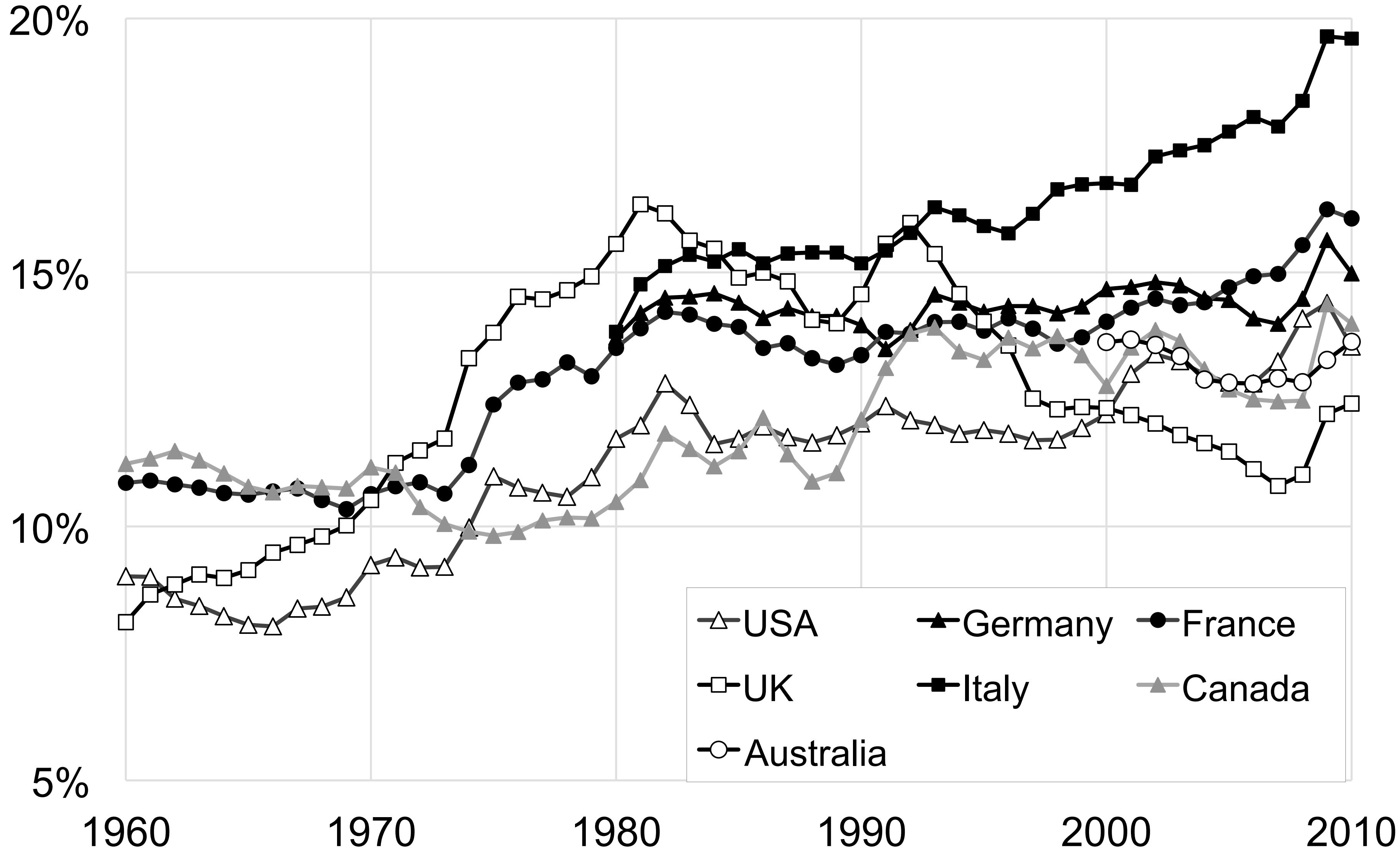
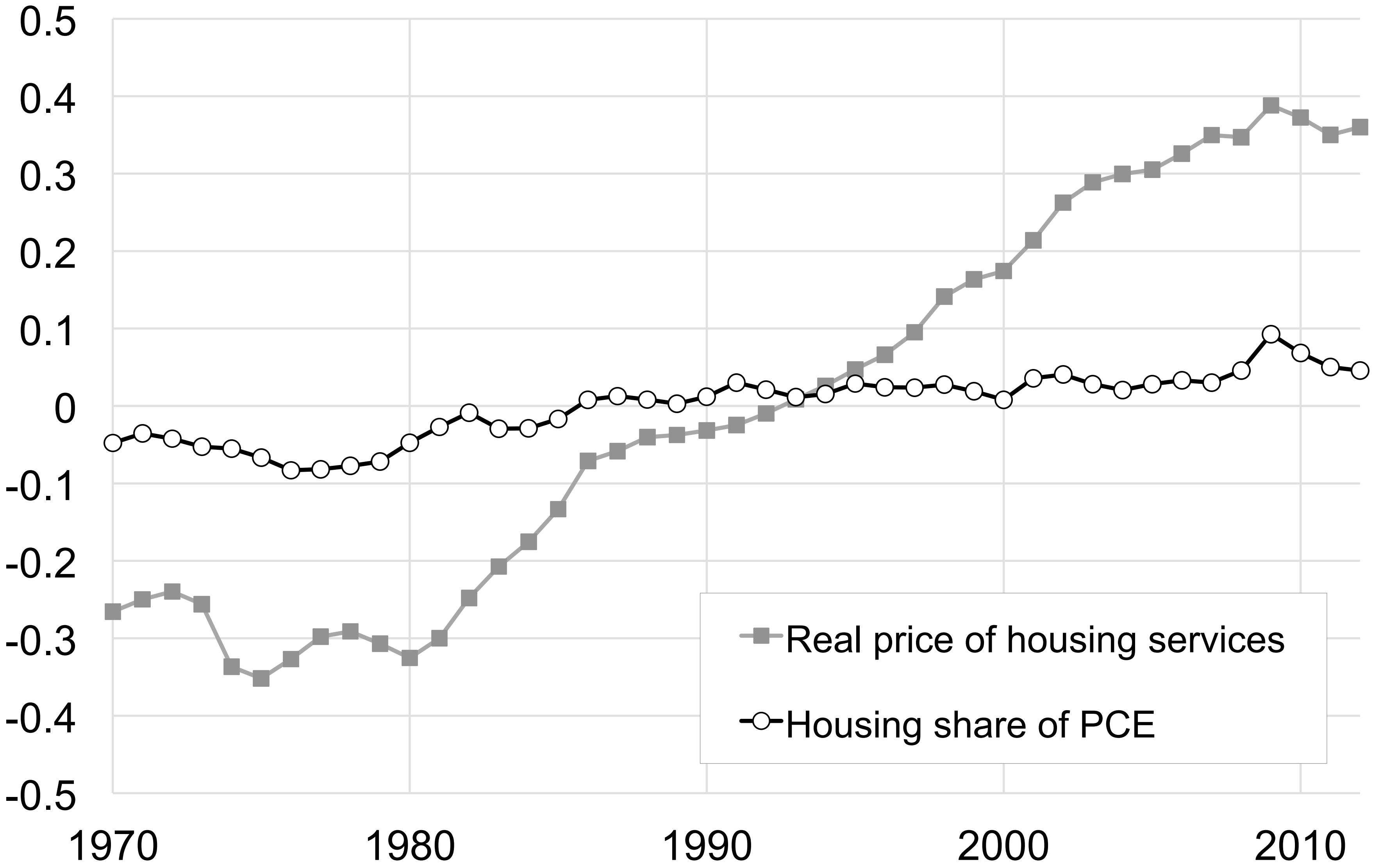


Figure 15: Real price of housing services versus housing services share of PCE (log scale), US



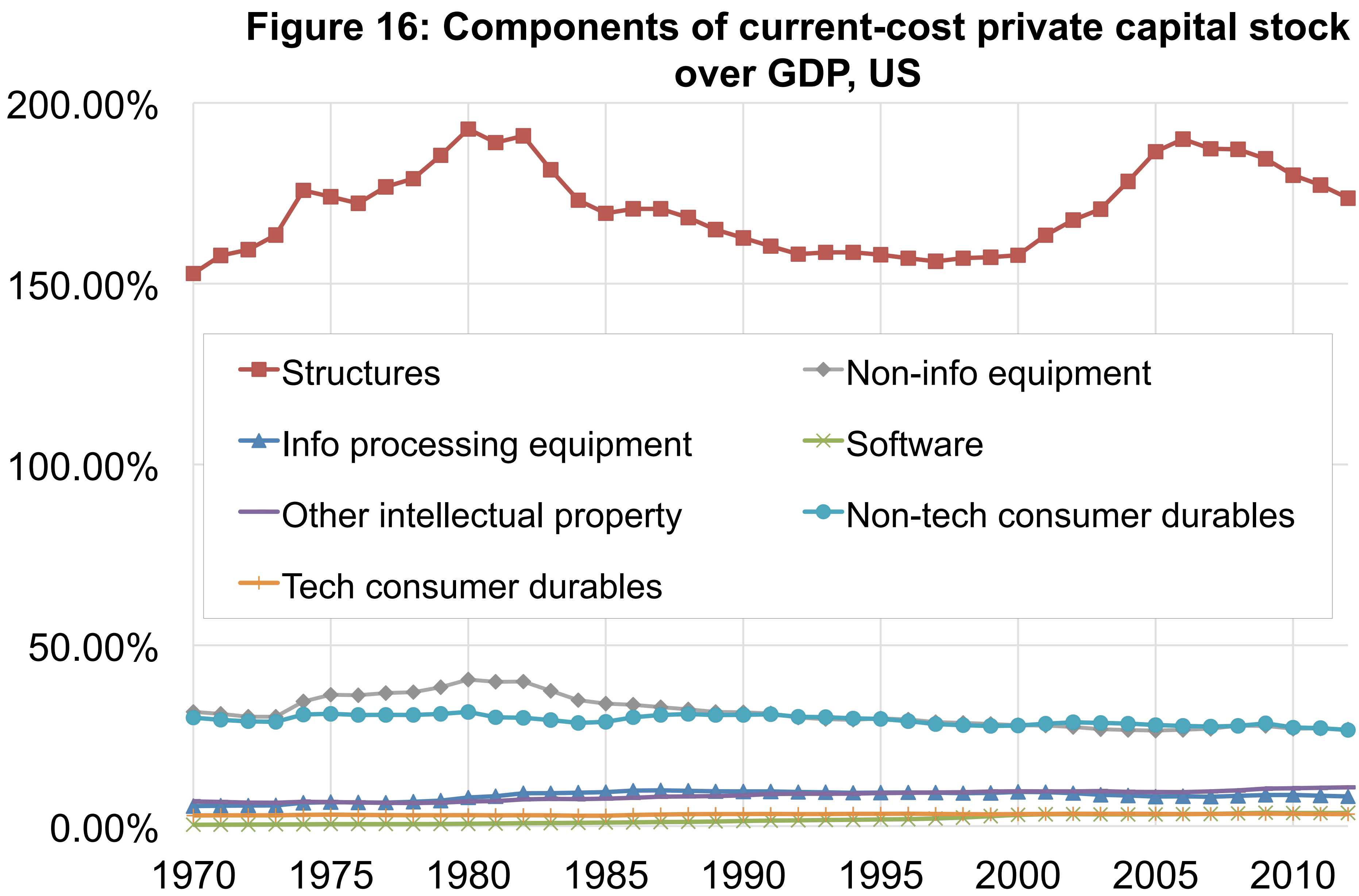


Figure 16: Components of current-cost private capital stock over GDP, US
200.00%
150.00%
100.00%
50.00%
0.00%
1970
1975
1980
1985
1990
1995
2000
2005
2010
Structures
Non-info equipment
Info processing equipment
Software
Other intellectual property
Non-tech consumer durables
Tech consumer durables

Figure 17: Selected technology-related components of current-cost private capital stock over GDP, US

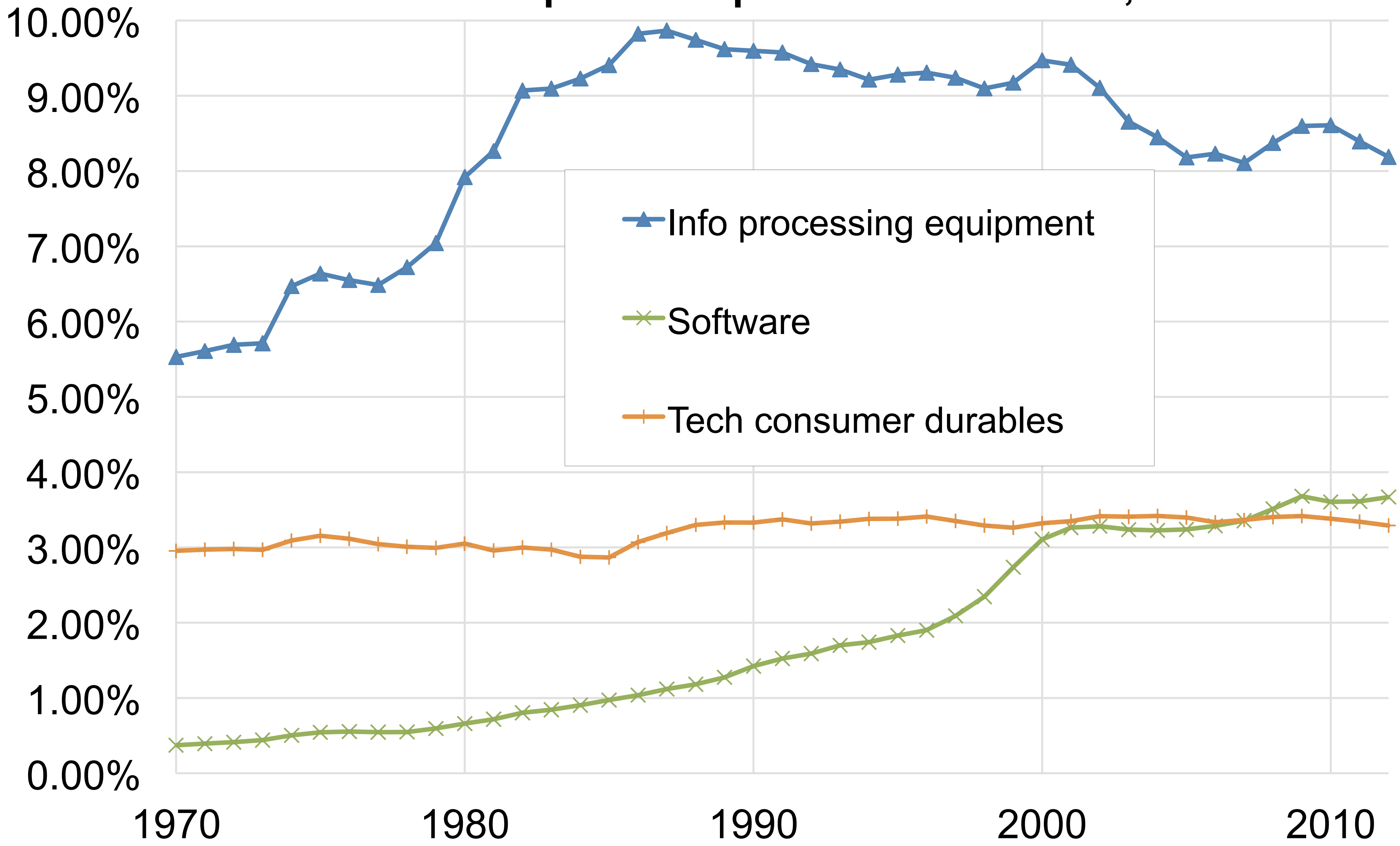


Figure 18: Real price of equipment versus equipment/GDP ratio (log scale), US

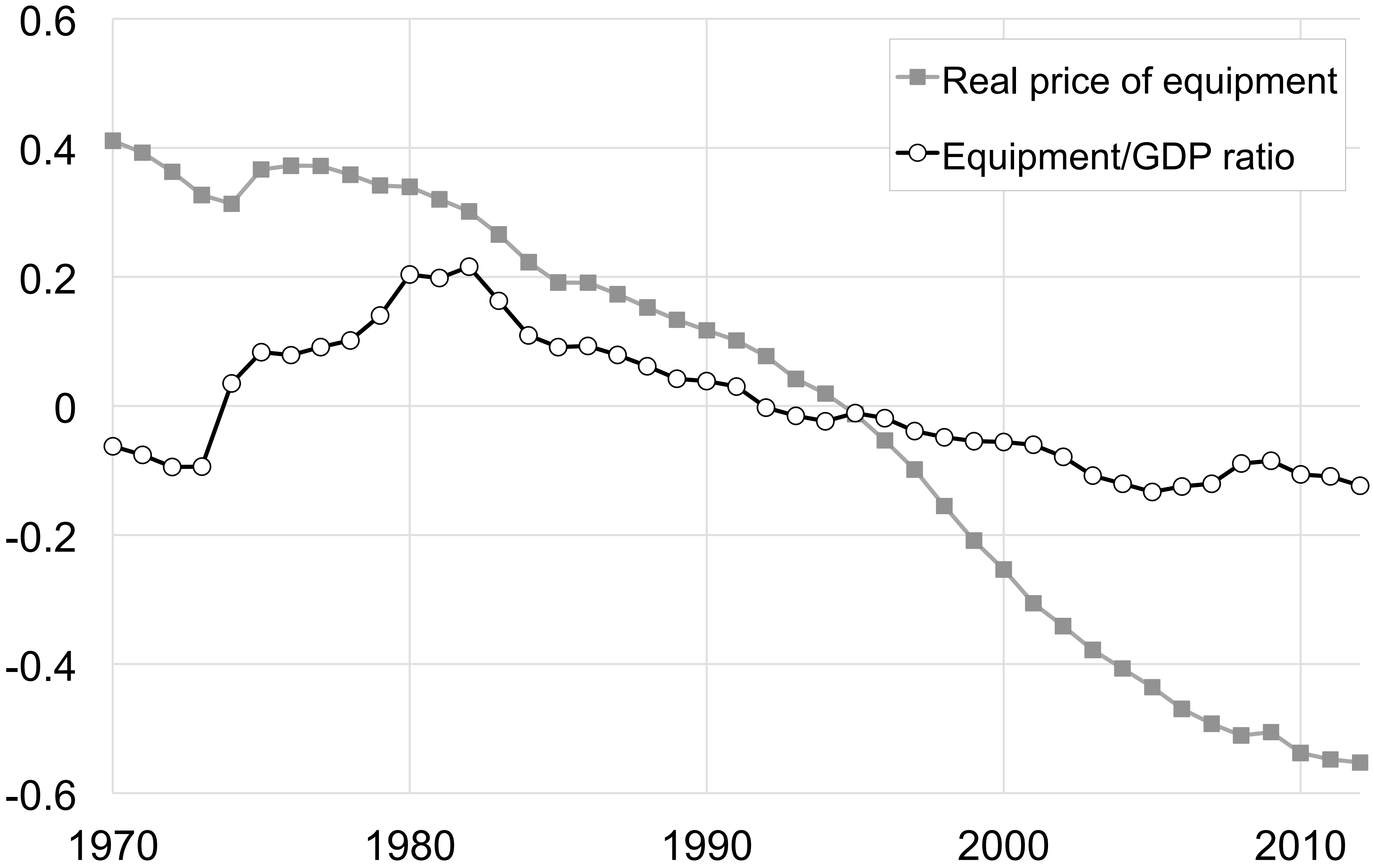


Figure 19: Real price of computers and peripheral equipment versus computer/GDP ratio (log scale), US

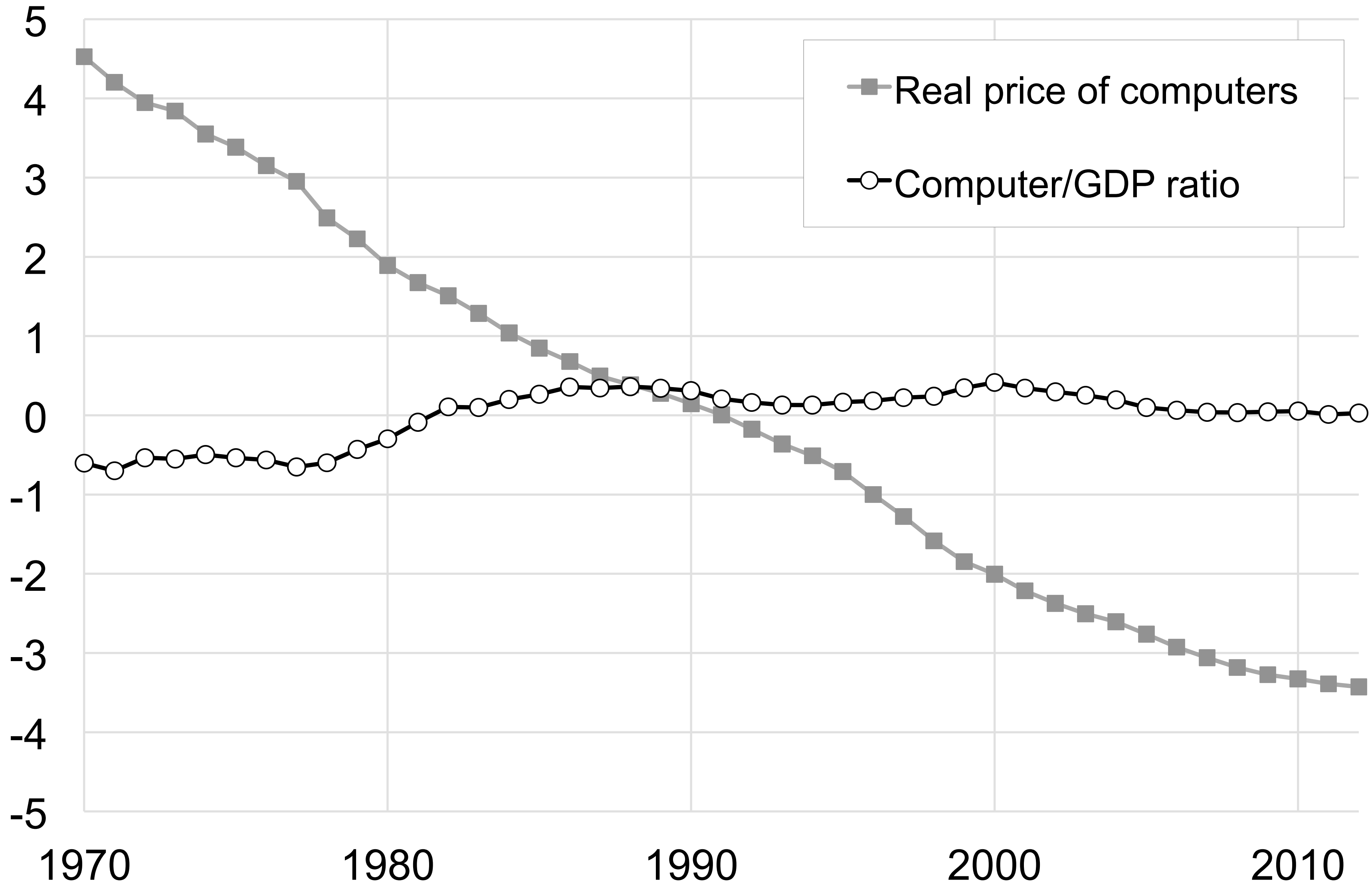


Figure 20: Real price of software versus software/GDP ratio (log scale), US