Chapter 6

Economic Inequality

6.1. Introduction

So far we have studied countries in their entirety. Economic growth is about changes in aggregate or average incomes. This is a good measure of a country's development, but it is far from being the only one. In this chapter, we begin to study a theme that recurs throughout the book: the analysis of the *distribution* of income, or wealth, among different groups in society. Economic growth that spreads its benefits equitably among the population is always welcome; growth that is distributed unequally needs to be evaluated not simply on the basis of overall change, but on the grounds of equity.

There are two reasons to be interested in the inequality of income and wealth distribution. First, there are philosophical and ethical grounds for aversion to inequality per se. There is no reason why individuals should be treated differently in terms of their access to lifetime economic resources.[1] It is, of course, possible to argue that people make choices—good and bad decisions—over the course of their lifetime for which only they are responsible. They are poor because "they had it coming to them." In some cases this may indeed be true, but in most cases the unequal treatment begins from day one. Parental wealth and parental access to resources can start two children off on an unequal footing, and for this fact there is little ethical defense. To hold descendants responsible for the sins of their ancestors is perhaps excessive. At the same time, we run into a separate ethical dilemma. To counteract the unequal treatment of individuals from the first day of their lives, we must deprive parents of the right to bequeath their wealth to their children. There may be no way to resolve this dilemma at a philosophical level.

Nevertheless, we can work toward a society with tolerable levels of inequality in everyday life. This goal reduces the dilemma in the preceding paragraph, because it reduces the scope for drastically unequal levels of accumulation (though of course it cannot entirely eliminate the problem). We cannot speak of development without a serious consideration of the problem of inequality.

[1] I make this statement assuming that there is no fundamental difference, such as the presence of a handicap or ailment, in the need for two people to have access to economic resources.

Second, *even if* we are uninterested in the problem of inequality at an intrinsic level, there is still good reason to worry about it. Suppose you simply care about overall growth, but find that inequality in income and wealth somehow reduce the possibilities of overall growth (or increase it; at this stage the direction of change is unimportant). Then you will care about inequality at what might be called a *functional level;* to you, inequality will be important not for its own sake, but because it has an impact on other economic features that you do care about.

In this book, we will pay attention to both the intrinsic and the functional features of inequality. To do this, we must first learn how to think about inequality at a conceptual level. This is the issue of *measurement,* which is the subject of this chapter. In Chapter 7, we will examine, both at an empirical and theoretical level, how inequality interacts with other economic variables, such as aggregate income and the growth of aggregate income.

6.2. What is economic inequality?

6.2.1. The context

At the level of philosophy, the notion of inequality can dissolve into an endless sequence of semantic issues. Ultimately, economic inequality is the fundamental disparity that permits one individual certain material choices, while denying another individual those very same choices. From this basic starting point begins a tree with many branches. Joao and Jose might both earn the same amount of money, but Joao may be physically handicapped while Jose isn't. John is richer than James, but John lives in a country that denies him many freedoms, such as the right to vote or travel freely. Shyamali earned more than Sheila did until they were both forty; thereafter Sheila did. These simple examples suggest the obvious: economic inequality is a slippery concept and is intimately linked to concepts such as lifetimes, personal capabilities, and political freedoms.[2]

Nevertheless, this is no reason to throw up our hands and say that *no* meaningful comparisons are possible. Disparities in personal income and wealth at any point of time, narrow though they may be in relation to the broader issues of freedom and capabilities, mean *something.* This statement is even more true when studying economic disparities *within* a country, because some of the broader issues can be regarded (at least approximately so) as affecting everyone in the same way. It is in this spirit that we study income and wealth inequalities: not because they stand for *all* differences, but because they represent an important component of those differences.

[2] On these and related matters, read the insightful discussions in Sen [1985].

6.2.2. Economic inequality: Preliminary observations

With the preceding qualifications in mind, let us turn to *economic* inequality: disparities in wealth or income. In this special case, some caveats need to be mentioned, even though we may not take them fully into account in what follows.

(1) Depending on the particular context, we may be interested in the distribution of current expenditure or income *flows*, the distribution of wealth (or asset *stocks*), or even the distribution of lifetime income. You can see right away that our preoccupation with these three possibilities leads us progressively from *short-term* to *long-term* considerations. Current income tells us about inequality at any one point of time, but such inequalities may be relatively harmless, both from an ethical point of view and from the point of view of their effects on the economic system, provided the inequality is temporary. To make this point clearly, consider the following example. Imagine two societies. In the first, there are two levels of income: $2,000 per month and $3,000 per month. In the second society, there are also two levels of income, but they are more dispersed: $1,000 per month and $4,000 per month. Let us suppose that the first society is completely mobile: people enter their working life at one of the two levels of income but stay there forever. In the second society, people exchange jobs every month, switching between the low-paid job and the high-paid job. These societies are obviously unrealistic caricatures, but they suffice to make the point. The first society shows up as more equal if income is measured at any one point in time, yet in terms of average yearly income, everyone earns the same in the second society.

Thus our notions of cross-sectional inequality at any one point in time must be tempered by a consideration of *mobility*. Whether each job category is "sticky" or "fluid" has implications for the true distribution of income. Often we are unable to make these observations as carefully as we would like, because of the lack of data, but that does not mean that we should be unaware of them.

(2) It may also be of interest to know (and we will get into this later in the book) not only *how much* people earn, but *how* it is earned. This is the distinction between *functional* and *personal* income distribution. Functional distribution tells us about the returns to different factors of production, such as labor (of different skills), capital equipment of various kinds, land, and so on. As you can imagine, this is only half the story. The next step is to describe how these different factors of production are owned by the individuals in society.

Figure 6.1 illustrates this process. Reading from left to right, the first set of arrows describes how income is generated from the production process. It is generated in varied forms. Production involves labor, for which wages

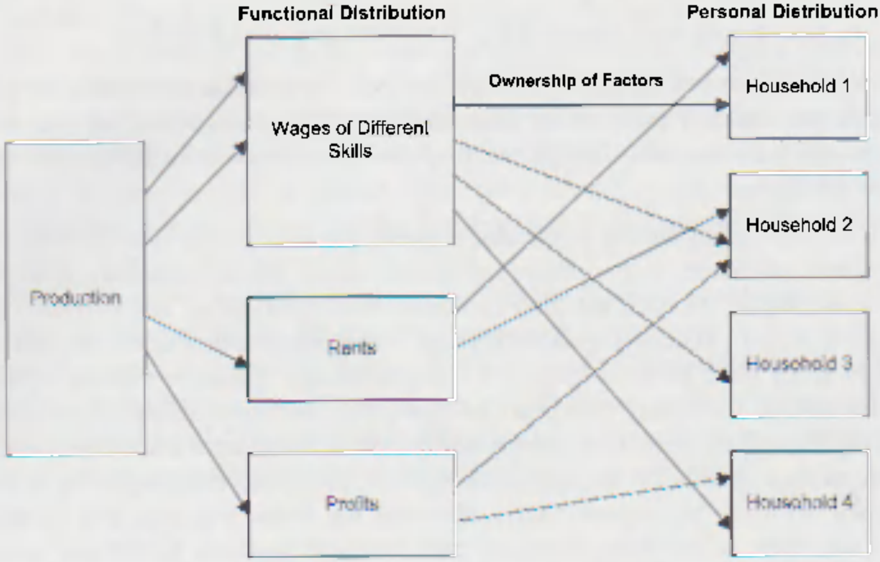

Figure 6.1. Functional and personal distribution of income.

are paid. It involves the use of land or capital equipment, for which rents are paid. It generates profits, which are paid out as well. Production also involves payments for various nonlabor inputs of production. These other inputs are in turn produced, so that in the ultimate analysis, all incomes that are generated can be classified under payments to labor of different skills, rents, and profits. The distribution of income under these various categories is the *functional distribution of income.*

The second set of arrows tells us how different categories of income are funneled to households. The direction and magnitude of these flows depend on who owns which factors of production (and how much of these factors). Households with only labor to offer (household 3 in the diagram, for instance) receive only wage income. In contrast, households that own shares in business, possess land to rent, and labor to supply (such as household 2) receive payments from all three sources. By combining the functional distribution of income with the distribution of factor ownership, we arrive at the *personal* distribution of income—a description of income flows to individuals or households, not factors of production.

You might well ask: why should we care about this two-step process? Isn't a direct knowledge of the personal distribution good enough for our analysis? The answer is that it isn't, and there are at least two good reasons for this. First, the understanding of income *sources* may well influence how we judge the outcome. Money that is received from charity or the welfare state may be viewed differently from the same amount received as income for work. Amartya Sen, in a closely related context, refers to this as the problem of "recognition" or self-esteem (see Sen [1975]):

> "Employment can be a factor in self-esteem and indeed in esteem by others. . . . If a person is forced by unemployment to take a job that he thinks is not appropriate for him, or not commensurate with his training, he may continue to feel unfulfilled and indeed may not even regard himself as employed."

Although there may not be much that we can do about this (so far as a theory of measurement goes), we should keep it in the back of our minds while we proceed to a final judgment about inequality.[3]

Second, and possibly more important, functional distribution tells us much about the relationship between inequality and other features of development, such as growth. Our understanding of how economic inequalities are created in a society necessitates that we understand both how factors are *paid* and how factors are *owned*.

The preceding discussion lays down a road map for our study of inequality. We look at economic inequalities from two angles. In this chapter, we put all sources of income into a black box and concentrate on the evaluation of income (or wealth or lifetime income) distributions. This part of the story is *normative*. All of us might like to see (other things being the same) an egalitarian society, but "egalitarian" is only a word: what does it mean when we are confronted with several alternative income distributions, which we must evaluate? How do we rank or order these distributions? This part of the chapter discusses how we measure inequality, or equivalently, how we rank alternative distributions with respect to how much inequality they embody.

With measurement issues out of the way, we proceed in Chapter 7 to a study of the economics of income distributions: how inequality evolves in society, the effects that it has on other features of economic development, such as output, employment and growth rates, and how these other features feed back in turn on income and wealth distributions. This part of the story is *positive*. Whether or not we like the notion of egalitarianism per se, inequality affects other features of development.

6.3. Measuring economic inequality

6.3.1. Introduction

If there is a great deal of disparity in the incomes of people in a society, the signs of such economic inequality are often quite visible. We probably

[3] Often, ingenious theories of measurement can go some way to resolve difficulties of this sort. For instance, it might matter for our measurement of literacy rate whether an literate person has access to *other* literate persons. On these matters, see Basu and Foster [1997].

know a society is very unequal when we see it. If two people are supposed to share a cake and one person has all of it, that's unequal. If they split 50–50, that's equal. We can even evaluate intermediate divisions (such as 30–70 and 40–60) with a fair amount of precision.

All that goes away, however, once we have more than two individuals and we try to rank intermediate divisions of the cake. Is it obvious how to compare a 20–30–50 division among three people with a 22–22–56 division? In such cases, and in even more complicated ones as well, it might be useful to try and "measure" inequality. This means that we develop or examine inequality indices that permit the ranking of income or wealth distributions in two different situations (countries, regions, points of time, and so on).

The question naturally arises: what are the properties that a "desirable" inequality index should satisfy? It is difficult to have complete unanimity on the subject, and there is none. If, to avoid controversy, we lay down only very weak criteria, then many inequality indices can be suggested, each consistent with the criteria, but probably giving very different results when used in actual inequality comparisons. If, on the other hand, we impose stricter criteria, then we sharply reduce the number of admissible indices, but the criteria loses wide approval.

As we will see, this problem is endemic, which is all the more reason to have a clear idea of what criteria lie behind a particular measure. Remember that by "believing" what a measure of inequality reports, you are identifying your intuitive notions of inequality with that particular measure. If you are a policy maker or an advisor, this form of identification can be useful or dangerous, depending on how well you understand the underlying criteria of measurement.

6.3.2. *Four criteria for inequality measurement*

Suppose that society is composed of n individuals.[4] We use the index i to stand for a generic individual; thus, $i = 1, 2, \ldots, n$. An *income distribution* is a description of how much income y_i is received by each individual i: $(y_1, y_2, \ldots, y_n)$.

We are interested in comparing the relative "inequality" of two income distributions. To this end, we need to capture some of our intuitive notions about inequality in the form of applicable criteria.

(1) *Anonymity principle.* From an ethical point of view, it does not matter *who* is earning the income. A situation where Debraj earns x and Rajiv earns

[4] In this section, we refer to "income" as the crucial variable whose inequality we wish to measure. You could replace this with wealth, lifetime income, and so on. Likewise, the recipient unit is called an individual. You could replace this by "household" or any other grouping that you might be interested in.

y should be viewed as identical (from the point of view of inequality) to one in which Debraj earns y and Rajiv earns x. Debraj may well be disgusted with this sort of change (if x happens to be larger than y), but it will be very difficult for him to persuade other people that the overall degree of inequality in his society has deteriorated because of this. Thus permutations of incomes among people should not matter for inequality judgments: this is the principle of *anonymity*. Formally, this means that we can always arrange our income distribution so that

$$y_1 \leq y_2 \leq \cdots \leq y_n,$$

which is the equivalent of arranging individuals so that they are ranked from poorest to richest.

(2) *Population principle*. Cloning the entire population (and their incomes) should not alter inequality. More formally, if we compare an income distribution over n people and another population of $2n$ people with the same income pattern repeated twice, there should be no difference in inequality among the two income distributions.[5] The population principle is a way of saying that population size does not matter: all that matters are the *proportions* of the population that earn different levels of income.

Criteria 1 and 2 permit us to view income distributions in a slightly different way. Typically, no data set is rich enough to tell us the incomes of every single individual in the country. Thus the data are often presented in the following way. There is a set of income *classes*, where each class typically is presented as a range of incomes; for example, "$100 per month or less," "$300–400," and so on.

Figure 6.2 illustrates this procedure using a hypothetical example. A population of people earn an income somewhere between zero and $1,000 in this example. The raw data are shown in the left panel of the figure. (You will almost never see data expressed like this for an actual population.) The anonymity principle tells us that we can number people in order of increasing income and no useful information is lost. The population principle tells us that it does not matter how many people there are; we may normalize everything to percentages. The right-hand panel gives us a common way to put together this information. Income classes are on the horizontal axis and the percentage of the population that falls into each income class is on the vertical axis. Neither the names of people nor the actual numbers in each income class matter.

[5] Warning: Cloning only one segment of the population while keeping the remainder unaltered may well affect our notions of inequality. Suppose that there are two incomes, $100 and $1,000. The population principle says that all income distributions are equally unequal provided the same percentage of people earn $100. If the *proportion* of people earning the low income changes, then inequality will, in general, be affected.

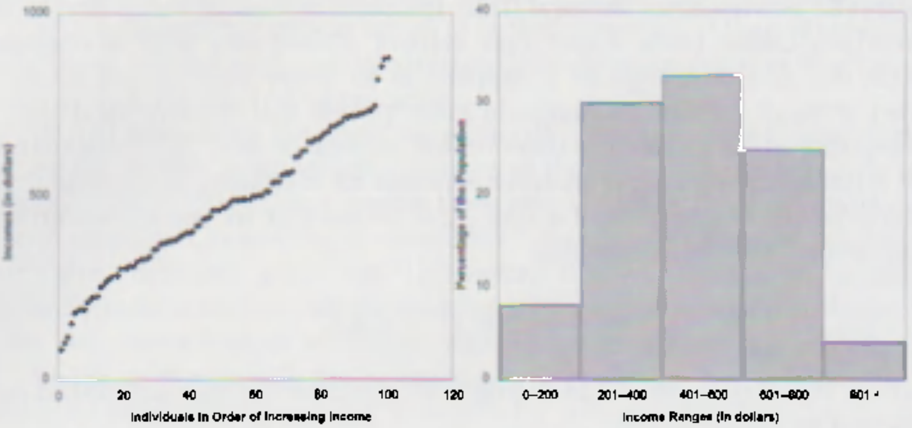

Figure 6.2. Income distribution arranged by income classes.

(3) *Relative income principle.* Just as population shares matter and the absolute values of the population itself do not, it is possible to argue that only *relative* incomes should matter and the absolute levels of these incomes should not. If one income distribution is obtained from another by scaling *everybody's* income up or down by the same percentage, then inequality should be no different across the two distributions. For instance, an income distribution over two people of ($1,000, $2,000) has the same inequality as ($2,000, $4,000), and this continues to be true if dollars are replaced by cruzeiros or yen. This is the relative income principle: it is tantamount to the assertion that income *levels,* in and of themselves, have no meaning as far as *inequality measurement* is concerned. Certainly, absolute incomes are important in our overall assessment of development, although the distinction between "absolute" and "relative" in the context of inequality measurement may not be that easy to draw.[6]

With the relative income principle in place, it is now possible to present data in a form that is even more stripped down. Both population and incomes can be expressed as shares of the total. The major advantage of this approach is that it enables us to compare income distributions for two countries that have different average income levels. Figure 6.3 shows how this is done with the very same hypothetical data that we used to generate Fig-

[6] Is it as easy to buy the relative income principle as the population principle? Not really. What we are after, in some sense, is the inequality of "happiness" or utility, however that may be measured. As matters stand, our presumption that inequality can be quantified at all forces us to make the assertion that the utilities of different individuals can be compared. (On the analytical framework of interpersonal comparability that is required to make systematic egalitarian judgments, see, for example, Sen [1970] and Roberts [1980].) However, the relative income principle needs more than that. It asserts that utilities are proportional to incomes. This is a strong assumption. We make it nevertheless because Chapter 8 will partially make amends by studying the effects of absolute income shortfalls below some poverty line.

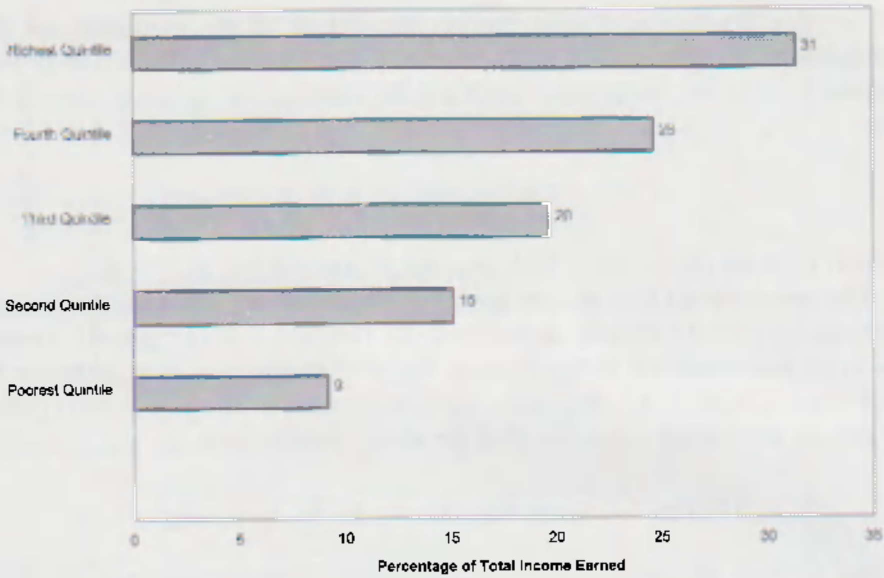

Figure 6.3. Income distribution by population and income shares.

ure 6.2. In Figure 6.3 we have divided the population into different equal-sized groups in order of poorest to richest. Because population size is unimportant in the measurement of inequality (by the population principle), using percentages is enough. We use quintiles (one could use deciles as well depending on the richness of the original data). For each quintile, we record the income share earned by that quintile of the population. Because people have been arrayed from poorest to richest, these income shares increase as we move from the first to the fifth quintile. The relative income principle tells us that income *shares* are all we need in the measurement of inequality.

(4) *The Dalton principle.* We are now in a position to state our final criterion for evaluating inequality. Formulated by Dalton [1920], [7] the criterion is fundamental to the construction of measures of inequality. Let $(y_1, y_2, \ldots, y_n)$ be an income distribution and consider two incomes y_i and y_j with $y_i \leq y_j$. A transfer of income from the "not richer" individual to the "not poorer" individual will be called a *regressive transfer*. The Dalton principle states that if one income distribution can be achieved from another by constructing a sequence of regressive transfers, then the former distribution must be deemed more unequal than the latter.

How far do these four criteria take us? Understanding this is the task of the next section. Before we do this, let us formally define an inequality measure. It is a rule that assigns a degree of inequality to each possible distribution of the national cake. In other words, it takes each income distribution

[7] See also Pigou [1912], after whom the principle is also called the Pigou–Dalton principle.

and assigns a value to it that can be thought of as the inequality of that distribution. A higher value of the measure signifies the presence of greater inequality. Thus an inequality index can be interpreted as a function of the form

$$I = I(y_1, y_2, \ldots, y_n)$$

defined over all conceivable distributions of income $(y_1, y_2, \ldots, y_n)$.

The requirement that the inequality measure satisfy the anonymity principle can be stated formally as follows: the function I is completely insensitive to all permutations of the income distribution $(y_1, y_2, \ldots, y_n)$ among the individuals $\{1, 2, \ldots, n\}$. Similarly, the requirement of the population principle can be translated as saying that for every distribution $(y_1, y_2, \ldots, y_n)$,

$$I(y_1, y_2, \ldots, y_n) = I(y_1, y_2, \ldots, y_n; y_1, y_2, \ldots, y_n),$$

so that cloning all members of the population and incomes has no effect. Thus by taking the lowest common multiple of the populations of any collection of income distributions, we can always regard each distribution as effectively having the same population size. The relative income principle can be incorporated by requiring that for every positive number λ,

$$I(y_1, y_2, \ldots, y_n) = I(\lambda y_1, \lambda y_2, \ldots, \lambda y_n).$$

Finally, I satisfies the Dalton transfer principle if, for every income distribution $(y_1, y_2, \ldots, y_n)$ and every transfer $\delta > 0$,

$$I(y_1, \ldots, y_i, \ldots, y_j, \ldots, y_n) < I(y_1, \ldots, y_i - \delta, \ldots, y_j + \delta, \ldots, y_n)$$

whenever $y_i \leq y_j$.

6.3.3. *The Lorenz curve*

There is a useful way to see what the four criteria of the previous section give us. Pictures often speak more than words, and in the context of inequality measurement, there is a nice diagrammatic way to depict the distribution of income in any society. The resulting graph is called the *Lorenz curve*, which is very often used in economic research and discussion; therefore, it is worthwhile to invest a little time in order to understand it.

Suppose we sort people in a population in increasing order of incomes. Figure 6.4 shows a typical Lorenz curve. On the horizontal axis, we depict *cumulative* percentages of the population arranged in increasing order of income. Thus points on that axis refer to the poorest 20% of the population,

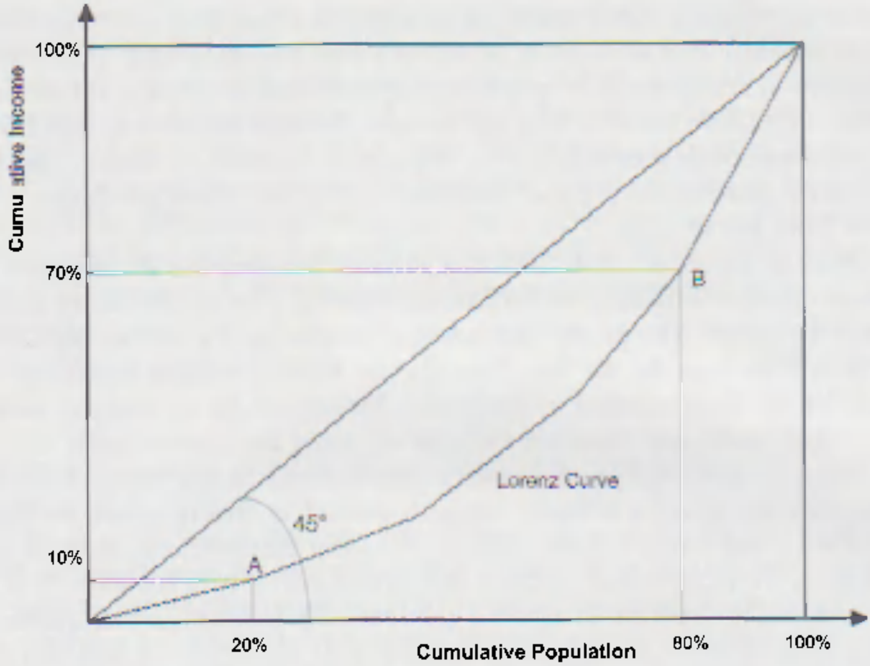

Figure 6.4. The Lorenz curve of an income distribution.

the poorest half of the population, and so on. On the vertical axis, we measure the percentage of national income accruing to any particular fraction of the population thus arranged. The point *A*, for example, corresponds to a value of 20% on the population axis and 10% on the income axis. The interpretation of this is that the poorest 20% of the population earns only 10% of overall income. Point *B*, on the other hand, corresponds to 80% on the population axis and 70% on the income axis. This point, therefore contains the information that the "poorest" 80% enjoy 70% of the national income. An equivalent way to describe this is from "above": the richest 20% have 30% of gross income for themselves. The graph that connects all these points is called the Lorenz curve.

Notice that the Lorenz curve begins and ends on the 45° line: the poorest 0% earn 0% of national income by definition and the poorest 100% is just the whole population, and so must earn 100% of the income. How would the Lorenz curve look like if everybody had the same income? Well, it would then coincide *everywhere* with the 45° line, that is, with the diagonal of the box. The poorest 10% (however selected) would then have exactly 10% of national income, whereas the richest 10% will also have the same 10%. In other words, any cumulative fraction of the population would share exactly that fraction of national wealth. Because the 45° line expresses the relationship $Y = X$, it *is* our Lorenz curve in this case. With increasing inequality, the

Lorenz curve starts to fall below the diagonal in a loop that is always bowed out to the right of the diagram; it cannot curve the other way. The slope of the curve at any point is simply the contribution of the person at that point to the cumulative share of national income. Because we have ordered individuals from poorest to richest, this "marginal contribution" cannot ever fall. This is the same as saying that the Lorenz curve can never get flatter as we move from left to right.

Thus in Figure 6.4, the "overall distance" between the 45° line and the Lorenz curve is indicative of the amount of inequality present in the society that it represents. The greater the extent of inequality, the further the Lorenz curve will be from the 45° line. Hence, even without writing down any formula for the measurement of inequality, we can obtain an intuitive idea of how much inequality there is by simply studying the Lorenz curve.

Some of the conceptual problems encoumtered in the measurement of inequality can also be brought out with the aid of this diagram. In Figure 6.5, the Lorenz curves of two different income distributions, marked $L(1)$ and $L(2)$, are represented. Because the second curve $L(2)$ lies entirely *below* the first one, it is natural to expect a good index to indicate greater inequality in the second case. Let's try to understand why this is so. The fact that $L(1)$ lies above $L(2)$ has the following easy interpretation: if we choose a poorest x% of the population (it does not matter what x you have in mind), then $L(1)$ always has this poorest x% earning at least as much as they do under $L(2)$. Thus regardless of which precise value of x you pick, the curve $L(1)$ is

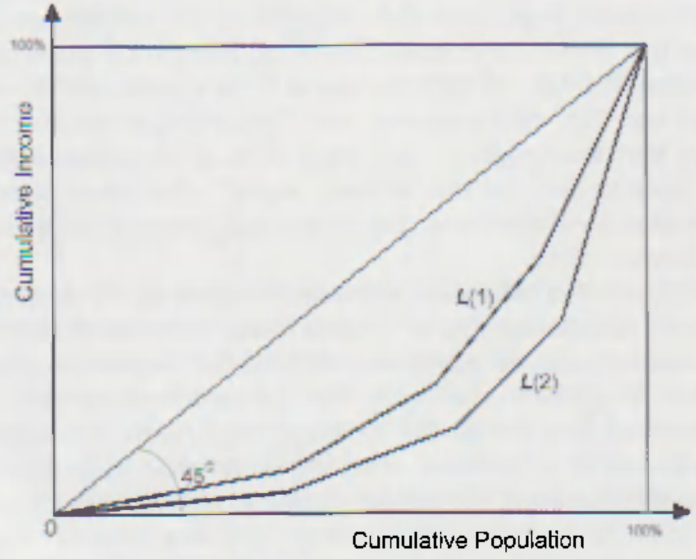

Figure 6.5. Using the Lorenz curve to make judgments.

always "biased" toward the poorest $x\%$ of the population, relative to $L(2)$. It stands to reason that $L(1)$ should be judged more equal than $L(2)$.

This criterion for inequality comparisons is known as the *Lorenz criterion*. It says that if the Lorenz curve of a distribution lies at every point to the right of the Lorenz curve of some other distribution, the former should be judged to be more unequal than the latter. Just as we required an inequality measure to be consistent with the criteria of the previous section, we require it to be consistent with this particular criterion. Thus an inequality measure I is *Lorenz-consistent* if, for every pair of income distributions $(y_1, y_2, \ldots, y_n)$ and $(z_1, z_2, \ldots, z_m)$,

$$I(y_1, y_2, \ldots, y_n) \geq I(z_1, z_2, \ldots, z_m)$$

whenever the Lorenz curve of $(y_1, y_2, \ldots, y_n)$ lies everywhere to the right of $(z_1, z_2, \ldots, z_m)$.

This is all very nice, but now confusion starts to set in. We just spent an entire section discussing four reasonable criteria for inequality comparisons and now we have introduced a fifth! Are these all independent restrictions that we have to observe? Fortunately, there is a neat connection between the four criteria of the previous section and the Lorenz criterion that we just introduced: *an inequality measure is consistent with the Lorenz criterion if and only if it is simultaneously consistent with the anonymity, population, relative income, and Dalton principles.*

This observation is very useful.[8] First, it shuts down our apparent expansion of criteria by stating that the earlier four are together exactly equivalent to the Lorenz criterion. Second, and more important, it captures our four criteria in one clean picture that gives us exactly their joint content. In this way we can summarize our verbal ethical criteria in simple graphical form.

The preceding observation is so central to our understanding of inequality that it is worth taking a minute to see why it is true. First, observe that the Lorenz curve automatically incorporates the principles of anonymity, population, and relative income, because the curve drops all information on income or population *magnitudes* and retains only information about income and population *shares*. What we need to understand is how the Dalton principle fits in. To see this, carry out a thought experiment. Pick any income distribution and transfer some resources from people, say from the fortieth population percentile, to people around the eightieth population percentile. This is a regressive transfer, and the Dalton principle says that inequality goes up as a result.

Figure 6.6 tells us what happens to the Lorenz curve. The thicker curve marks the original Lorenz curve and the thinner curve shows us the Lorenz

[8] For a useful discussion of the history of this result, see the survey by Foster [1985].

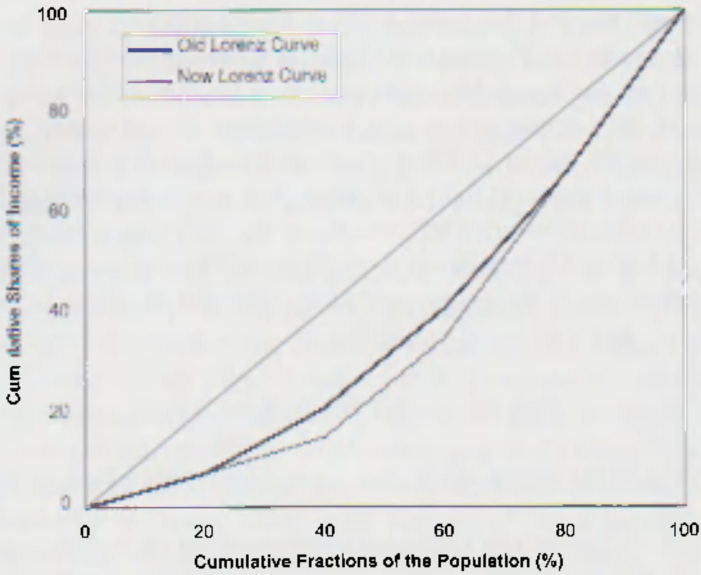

Figure 6.6. The Dalton principle and the Lorenz criterion.

curve after the transfer of resources. What about the new curve? Well, nothing was disturbed until we get close to the fortieth percentile, and then, because resources were transferred away, the share of this percentile *falls*. The new Lorenz curve therefore dips below and *to the right* of the old Lorenz curve at this point. What is more, it stays below for a while. Look at a point around the sixtieth population percentile. The income shares here are reduced as well, even though the incomes of people around this point were not tampered with. The reason for the reduction is that Lorenz curves plot *cumulative* population shares on the horizontal axis and their *cumulative* income share on the vertical axis. Because people from the fortieth percentile were "taxed" for the benefit of the eightieth percentile, the new share at the sixtieth percentile population mark (and indeed, at all percentiles between forty and eighty) must also be lower than the older share. This state of affairs persists until the eightieth percentile comes along, at which point the overall effect of the transfer vanishes. At this stage the *cumulative* shares return to exactly the level at which they were before. In other words, the Lorenz curves again coincide after this point. In summary, the new Lorenz curve is bowed to the right of the old (at least over an interval), which means that the Lorenz criterion mirrors the Dalton principle; that is, they agree.

The converse comparison is true as well: if two Lorenz curves are comparable according to the Lorenz criterion, as in the case of $L(1)$ and $L(2)$ in Figure 6.5, then it *must* be possible to construct a set of disequalizing transfers leading from $L(1)$ to $L(2)$. We leave the details to an exercise at the end of this chapter.

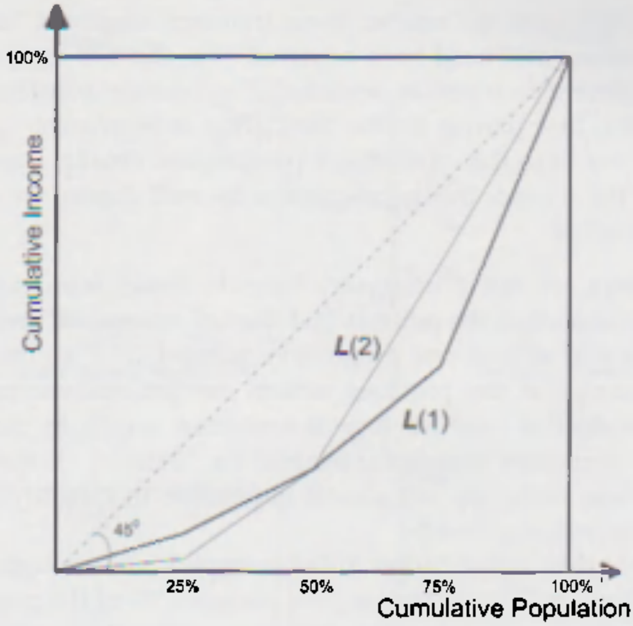

Figure 6.7. Ambiguous comparisons: A Lorenz crossing.

At this point, it looks like we are all set. We have a set of criteria that has a clear diagrammatic reformulation. It appears we can compare Lorenz curves using these criteria, so there is apparently no more need to make a fuss about inequality measurement. Unfortunately, matters are a bit more complicated. Two Lorenz curves can *cross.*

Figure 6.7 illustrates a Lorenz crossing. There are two income distributions that are represented by the Lorenz curves $L(1)$ and $L(2)$ in the diagram. Observe that neither Lorenz curve is uniformly to the right of the other. For two income distributions that relate to each other in this fashion, the Lorenz criterion does not apply. By the equivalence result discussed previously, it follows that our four principles cannot apply either, but what does it mean for these criteria "not to apply"? It means that we *cannot* go from one distribution to the other by a sequence of Dalton regressive transfers. Put another way, there must be *both* "progressive" and "regressive" transfers in going from one distribution to the other. The following example illustrates this point.

Example. Suppose that society consists of four individuals who earn incomes of 75, 125, 200, and 600. Now consider a second income distribution, given by (25, 175, 400, 400). Compare the two. We can "travel" from the first distribution to the second in the following manner. First transfer 50 from the first person to the second: this is a regressive transfer. Next transfer 200 from the fourth person to the third: this is a progressive transfer. We have arrived at the second distribution. Of course, these transfers are just a "construction" and not something that need have occurred (e.g., the two distributions may be for two different four-person societies). Try another construction. Transfer 50 from the first person to the third: This is regressive. Transfer now 150 from the fourth to the third: this is progressive. Finally, transfer 50 from the fourth to the second: this is progressive as well. Again, we arrive at the second distribution.

Hence, there are many imaginary ways to travel from the first to the second distribution, but the point is that they *all* necessarily involve at least one regressive and at least one progressive transfer. (Try it.) In other words, the four principles of the previous section are just not enough to permit a comparison. In this case we *have* to somehow weigh in our minds the "cost" of the regressive transfer(s) against the "benefit" of the progressive transfer(s). These trade-offs are almost impossible to quantify in a way so that everybody will approve.[9]

What about the Lorenz curves in the example? Sure enough, they mirror the complications of the comparison. The poorest 25% of the population earn 7.5% of the income in the first distribution and only 2.5% of the income in the second. The opposite comparison holds when we get to the poorest 75% of the population, who enjoy only 40% of the total income under the first distribution, but 60% of the income under the second distribution.

Now go back and look at Figure 6.7 once again. You can see that $L(1)$ and $L(2)$ are precisely the Lorenz curves for the two distributions in this example.

Despite these ambiguities, Lorenz curves provide a clean, visual image of the overall distribution of income in a country. Figure 6.8 provides several examples of Lorenz curves for different countries. By looking at these figures, you can get a sense of income inequalities in different parts of the world, and with a little mental superimposition of any two diagrams you can compare inequalities across two countries.

6.3.4. *Complete measures of inequality*

Lorenz curves provide a pictorial representation of the degree of inequality in a society. There are two problems with such a representation. First, policy makers and researchers are often interested in summarizing inequal-

[9] Shorrocks and Foster [1987] argued for a fifth principle, which they call transfer sensitivity. This principle tries to compare progressive transfers at the lower end of the income distribution with regressive transfers at the upper end, arguing that if both involve the same size transfer, then the former should be "ethically allowed" to outweigh the latter: inequality should come down under the composite transfer. This principle further broadens the scope of comparison, but is still not enough to rule out ambiguities.

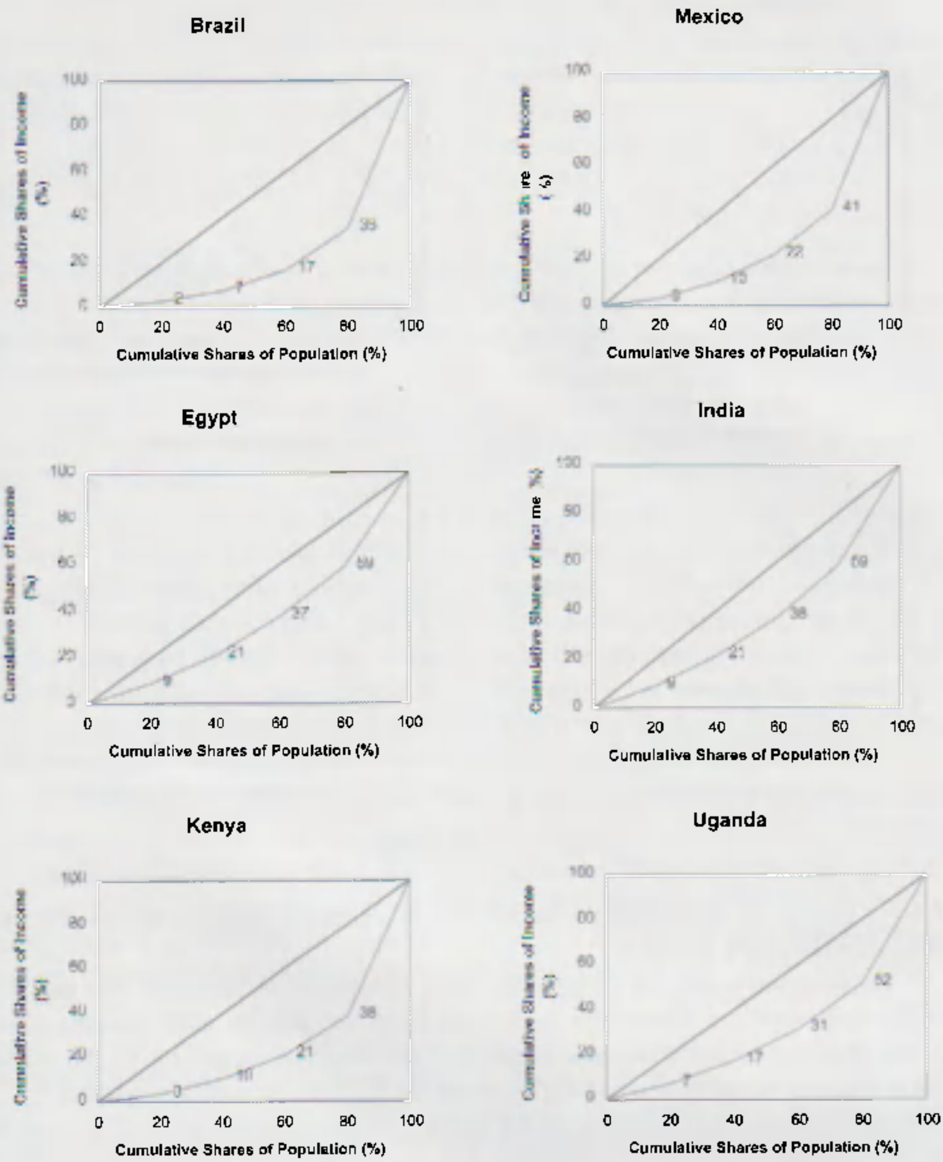

Figure 6.8. Lorenz curves for different countries. Source: Deininger–Squire data base; see Deininger and Squire [1996a].

ity by a *number*, something that is more concrete and quantifiable than a picture. Second, when Lorenz curves cross, they cannot provide useful inequality rankings. Thus an inequality measure that spits out a number for every conceivable income distribution can be thought of as a *complete ranking* of income distributions. As we will see, this completeness does not come

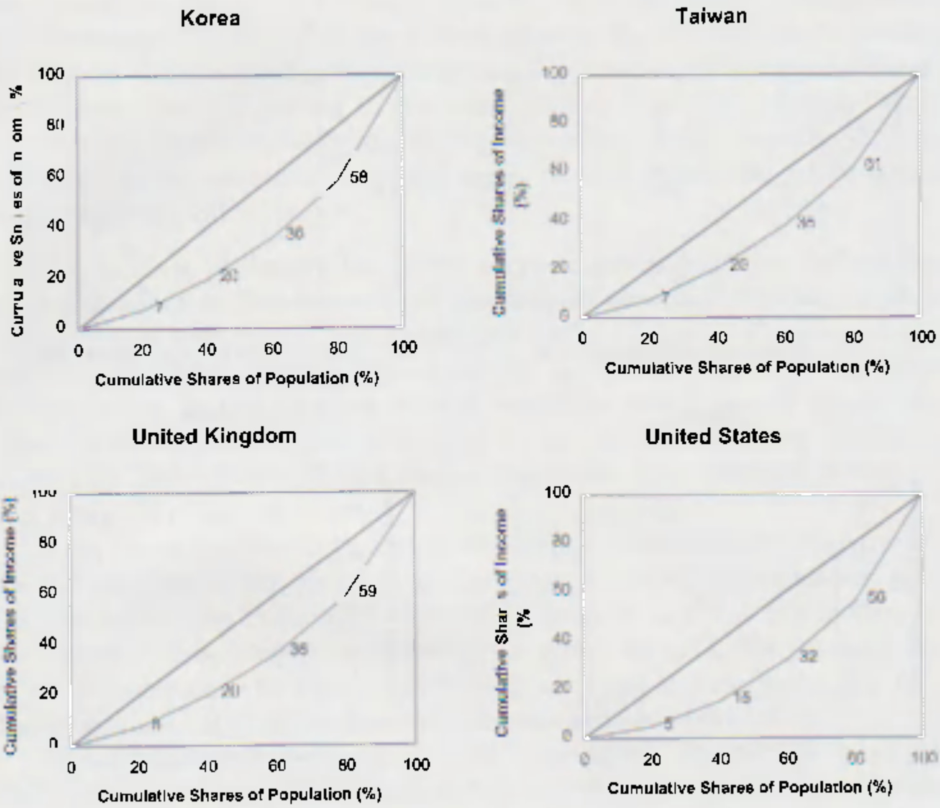

Figure 6.8. (Continued).

free of charge: it means that in some situations, inequality measures tend to disagree with one another.

We now survey some commonly used inequality measures.[10] We use the following notation. There are m distinct incomes, and in each income class j, the number of individuals earning that income is denoted by n_j. Thus the total number of people n is simply equal to $\sum_{j=1}^{m} n_j$, where the symbol $\sum_{j=1}^{m}$ henceforth denotes the *sum* over the income classes 1 through m. The *mean* μ of any income distribution is simply average income, or total income divided by the total number of people. Thus

$$\mu \equiv \frac{1}{n}\sum_{j=1}^{m} n_j y_j.$$

The following (complete) measures of inequality are often used.

[10] See Sen [1997] for a discussion of these and other measures, and for a comprehensive overall treatment of the subject of economic inequality.

(1) *The range*. This value is given by the difference in the incomes of the richest and the poorest individuals, divided by the mean to express it independently of the units in which income is measured. Thus the range R is given by

$$R = \frac{1}{\mu}(y_m - y_1). \tag{6.1}$$

Quite obviously, this is a rather crude measure. It pays no attention, whatsoever, to people between the richest and the poorest on the income scale. In particular, it fails to satisfy the Dalton principle, because, for example, a small transfer from the second poorest to the second most rich individual will keep the measure unchanged. However, the range is often used as a crude, though useful, measure when detailed information on income distribution is missing.

(2) *The Kuznets ratios*. Simon Kuznets introduced these ratios in his pioneering study of income distributions in developed and developing countries. These ratios refer to the share of income owned by the poorest 20 or 40% of the population, or by the richest 10%, or more commonly to the *ratio* of the shares of income of the richest x% to the poorest y%, where x and y stand for numbers such as 10, 20, or 40. The ratios are essentially "pieces" of the Lorenz curve and, like the range, serve as a useful shorthand in situations where detailed income distribution data are missing.

(3) *The mean absolute deviation*. This is our first measure that takes advantage of the entire income distribution. The idea is simple: inequality is proportional to distance from the mean income. Therefore, simply take all income distances from the average income, add them up, and divide by total income to express the average deviation as a fraction of total income. This means that the mean absolute deviation M is defined as

$$M = \frac{1}{\mu n}\sum_{j=1}^{m} n_j |y_j - \mu|, \tag{6.2}$$

where the notation $|\cdot|$ stands for the absolute value (neglecting negative signs). Although M looks promising as a measure of inequality that takes into account the overall income distribution, it has one serious drawback: it is often insensitive to the Dalton principle. Suppose that there are two people with incomes y_j and y_k, such that y_j is below the mean income of the population and y_k is above the mean income of the population. Then a regressive income transfer from y_j to y_k certainly raises inequality as measured by M. This is obvious from the formula, because the distance of both y_j and y_k goes up and no other distance is altered, so M unambiguously rises. However, the Dalton principle is meant to apply to *all* regressive transfers, not just those from incomes below the mean to incomes above the mean. For example, take any two incomes y_j and y_k that are both above the mean, and make a transfer from the lower of the two, say y_j, to the other (higher) one. Clearly, if the transfer is small enough so that both incomes stay above the mean after the transfer, there will be no difference in the sum of the absolute difference from mean income. The mean absolute deviation will register no change in such a case, and so fails the Dalton principle. We must conclude that using as it does the entire income distribution, the mean absolute deviation has no compensatory features as a quick estimate and is therefore a bad measure of inequality.

(4) *The coefficient of variation.* One way to avoid the insensitivity of the mean absolute deviation is by giving more weight to larger deviations from the mean. A familiar statistical measure that does just this is the standard deviation (see Appendix 2), which squares all deviations from the mean. Because the square of a number rises more than proportionately to the number itself, this is effectively the same as attaching greater weight to larger deviations from the mean. The coefficient of variation (C) is just the standard deviation divided by the mean, so that only relative incomes matter. Thus

$$C = \frac{1}{\mu n}\sqrt{\sum_{j=1}^{m} n_j (y_j - \mu)^2}. \tag{6.3}$$

The measure C, it turns out, has satisfactory properties. It satisfies all four principles and so it is Lorenz-consistent. In particular, it always satisfies the Dalton transfer principle. Consider a transfer from j to k, where $y_j \leq y_k$. This implies a transfer from a smaller number [i.e., $(y_j - \mu)$] to a larger one [i.e., $(y_k - \mu)$], which increases the *square* of the larger number by more than it decreases the square of the smaller number. The net effect is that C invariably registers an increase when such a regressive transfer is made. You should check by trying out various examples that this is always the case.

(5) *The Gini coefficient.* We now come to a measure that is widely used in empirical work: the Gini coefficient. The Gini approach starts from a base that is fundamentally different from measures such as M and C. Instead of taking deviations from the mean income, it takes the difference between *all* pairs of incomes and simply totals the (absolute) differences. It is as if inequality is the sum of all pairwise comparisons of "two-person inequalities" that can conceivably be made. The Gini coefficient is normalized by dividing by population squared (because all pairs are added and there are n^2 such pairs) as well as mean income. In symbols, the Gini coefficient G is given by

$$G = \frac{1}{2n^2\mu}\sum_{j=1}^{m}\sum_{k=1}^{m} n_j n_k |y_j - y_k|. \tag{6.4}$$

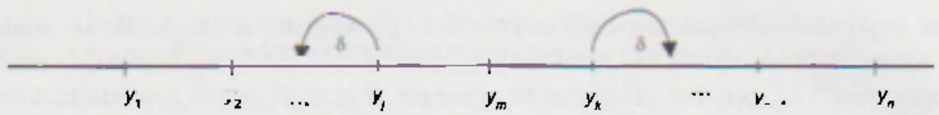

Figure 6.9. The Lorenz consistency of the Gini coefficient.

The double summation sign signifies that we first sum over all js, holding each i constant, and then sum over all the is. This is like summing all pairs of income differences (weighted by the number of such pairs, $n_j n_k$). Notice that because each $|y_j - y_k|$ is counted twice (again as $|y_k - y_j|$), the whole expression is finally divided by 2 as well as by the population and income normalizers.

The Gini coefficient has pleasing properties. It satisfies all four principles and is therefore Lorenz-consistent, just like the coefficient of variafion. Figure 6.9 shows us why the Gini coefficient is consistent with the Lorenz criterion. In this figure, we arrange everybody's incomes from lowest to highest. Now take two incomes, say y_j and y_k, with $y_j < y_k$, and transfer some small amount[11] δ from y_j to y_k. Figure 6.9 shows us how these two incomes change. Now let us see how the Gini coefficient has altered as a result of this regressive transfer. All we have to do is see the change in those pairs in which j or k figure. Consider incomes to the left of y_j. Because y_j has come down, the difference between these incomes and y_j has narrowed by δ. This narrowing is exactly counterbalanced by the fact that y_k has gone up by the same amount, so the distance between y_k and incomes to the left of y_j has gone up by an equal amount. The same argument holds for incomes to the right of y_k: the distance between them and y_k narrows, but the distance to y_j goes up by the same amount, so all these effects cancel. This leaves us with incomes between y_j and y_k. However, the pairwise distance between these incomes and *both* y_j and y_k has gone up. So has the distance between y_j and y_k. Thus the overall effect is an increase in the Gini coefficient. This shows why the Gini coefficient is Lorenz-consistent.

There is another interesting property of the Gini coefficient that ties it very closely indeed to the Lorenz curve. Recall that the more "bowed out" the Lorenz curve, the higher is our intuitive perception of inequality. It turns out that the Gini coefficient is precisely the ratio of the *area* between the Lorenz curve and the 45° line of perfect equality, to the *area* of the triangle below the 45° line.

We have thus surveyed five indexes. Of these, the first two are very crude but nevertheless useful indicators of inequality when detailed data are unavailable. The third should not be used. Finally, both the coefficient

[11] We take δ small so as to preserve the ranking of individuals in ascending order of income. The argument for larger values of δ follows by breaking up the transfer into smaller pieces and applying the logic in the text.

of variation (C) and the Gini coefficient (G) appear to be perfectly satisfactory indexes, going by our four principles (or what is equivalent, Lorenz consistency),[12] but this gives rise to a puzzle. If both C and G are satisfactory in this sense, why use both measures? Why not just one?

This brings us back full circle to Lorenz crossings. We have just seen that both C and G are Lorenz-consistent. This means that when Lorenz curves *can* be compared, both C and G give us exactly the same ranking, because they both agree with the Lorenz criterion. The problem arises when two Lorenz curves cross. In that case, it is possible for the Gini coefficient and the coefficient of variation to give contradictory rankings. This is nothing but a reflection of the fact that our intuitive sense of inequality is essentially incomplete. In such situations, we should probably not rely entirely on one particular measure of inequality, but rely on a whole set of measures. It may be a good idea to simply study the two Lorenz curves as well.

As a hypothetical example, consider two societies, each consisting of only three persons. Let the distribution of income in the two societies be (3, 12, 12) and (4, 9, 14), respectively. You can easily check that for the first of our hypothetical societies, the coefficient of variation is 0.27, whereas it is 0.26 for the second. Using C as an index, therefore, we reach the conclusion that the first society is more unequal than the second. However, if we calculate the Gini coefficient, the values come out to be 0.22 and 0.25, respectively. On the basis of the latter measure, therefore, inequality seems to be higher in the second society compared to the first.[13]

To be sure, this isn't just a hypothetical possibility. Such contradictory movements of inequality indexes occur in real life as well. Consider, for instance, the study by Weisskoff [1970] on inequality variations in Puerto Rico, Argentina, and Mexico during the 1950s. Table 6.1, put together from Weisskoff's study by Fields [1980], illustrates the ambiguities that arise.

The table is remarkable in its varied movements of inequality measures. In each of the three countries, there is some ambiguity. In Puerto Rico, for instance, *both* the poorest 40% and the richest 5% of the population lost income

[12] Of course, other measures are in use as well. There is the use of *log variance* as an inequality measure, which is just the standard deviation of the logarithm of incomes. Although it is easy to compute and use, the log variance unfortunately disagrees with the Dalton principle in some cases. Another measure, introduced to inequality evaluation by Henri Theil and known as the *Theil index*, is derived from entropy theory. Although it looks bizarre at first, it turns out to be the *only* measure that satisfies the four principles and a convenient decomposability principle that permits us to separate overall inequality into between-group and within-group components (Foster [1983]). This makes the Theil index uniquely useful in situations where we want to decompose inequality into various categories, for example, inequality within and across ethnic, religious, caste, occupational, or geographical lines.

[13] Warning: There is no connection between a value of, say, 0.25 achieved by the Gini coefficient compared to the same number achieved by C. That's like comparing apples and oranges. All this example is doing is contrasting different trends in the movements of these indexes as the distribution of income changes.

Table 6.1. Changes in inequality in Puerto Rico, Argentina, and Mexico.

Country/date	*Gini*	*Coeff. of variation*	*Income share of richest 5% (%)*	*Income share of poorest 40% (%)*
Puerto Rico				
1953	0.415	1.152	23.4	15.5
1963	0.449↑	1.035↓	22.0↓	13.7↑
Argentina				
1953	0.412	1.612	27.2	18.1
1959	0.463↑	1.887↑	31.8↑	16.4↑
1961	0.434↓↑	1.605↓↓	29.4↓↑	17.4↓↑
Mexico				
1950	0.526	2.500	40.0	14.3
1957	0.551↑	1.652↓	37.0↓	11.3↑
1963	0.543↓↑	1.380↓↓	28.8↓↓	10.1↑↑

Source: Fields [1980]. Note: First arrow indicates a change in inequality from the previous observation; the second arrow indicates the change in inequality from two observations ago.

share, a clear sign that the Lorenz curve has crossed. This doesn't *necessarily* mean that the Gini coefficient and the coefficient of variation disagree, but they do in fact. In the case of Argentina, there is no evidence from the income shares of the richest and poorest that the Lorenz curves have crossed, but they must have, at least over the period 1953–61. Do you see why? Finally, look at Mexico for the period 1957–63. In this case, both the Gini coefficient and the coefficient of variation agree, but it's also clear from the movement of income shares that the Lorenz curves have crossed (check this). It should be abundantly clear, therefore, that unless we have a clear case where a Lorenz comparison can be made, we should consult a variety of inequality measures before making a judgment.

Clearly we have a dilemma here: the result of our comparison is sensitive to the choice of the index, but we have no clear intuitive reason to prefer one over the other. There are two ways out of this dilemma. The first, as we said before, is to examine our *notion* of inequality more closely and to come up with stricter criteria after such introspection. The result, as was pointed out, will inevitably be subjective and controversial. The second escape is to realize that human thought and ideas abound with *incomplete* orderings: everyone agrees that Shakespeare is a greater writer than the Saturday columnist in the local newspaper; however, you and I might disagree whether he is greater than Tagore or Tolstoy, and even I may not be very sure *myself*. Relative inequality, like relative literary strength, may be perfectly discernible some of the time and difficult to judge in other cases. We can learn to live with that. If a society manages to significantly increase economic fairness and humane distribution among its members, then this fact will be captured in every reasonable inequality index, and we will not have to quibble over technicalities! It pays, however, to be aware of the difficulties of measurement.

In the next chapter, we go back to economics instead of plain measurement. Our goal will be to relate inequality to other features of the development process.

6.4. Summary

In this chapter, we studied the measurement of *inequality* in the distribution of wealth or incomes. We argued that there are two reasons to be interested in inequality: the *intrinsic*, in which we value equality for its own sake and therefore regard inequality reduction as an objective in itself, and the *functional*, in which we study inequality to understand its impact on *other* features of the development process.

As a prelude to the study of measurement, we recognized that there were several conceptual issues. For instance, inequality in incomes may be compatible with overall equality simply because a society might display a high degree of *mobility*: movement of people from one income class to another. We also paid attention to the *functional distribution* of income as opposed to the *personal distribution* of income: *how* income is earned may have just as much social value as *how much* is earned.

With these caveats in mind, we then introduced four criteria for inequality measurement: (1) the *anonymity principle* (names do not matter), (2) the *population principle* (population size does not matter as long as the *composition* of different income classes stay the same in percentage terms), (3) the *relative income principle* (only relative incomes matter for the measurement of inequality, and not the absolute amounts involved), and (4) the *Dalton transfer principle* (if a transfer of income is made from a relatively poor to a relatively rich person, then inequality, however measured, registers an increase). It turns out that these four principles create a ranking of income distribution identical to that implied by the *Lorenz curve*, which displays how cumulative shares of income are earned by cumulatively increasing fractions of the population, arranged from poorest to richest.

However, the ranking is not complete. Sometimes two Lorenz curves cross. In such situations the four principles are not enough to make an unequivocal judgment about inequality. We argued that in this sense, our notions of inequality are fundamentally incomplete, but that forcing an additional degree of completeness by introducing more axioms may not necessarily be a good idea.

Complete measures of inequality do exist. These are measures that assign a degree of inequality (a number) to *every* conceivable income distribution, so they generate complete rankings. We studied examples of such measures that are popularly used in the literature: the *range*, the *Kuznets ratios*, the *mean absolute deviation*, the *coefficient of variation*, and the *Gini coefficient*. Of these measures, the last two are of special interest in that they agree fully with our four principles (and so agree with the Lorenz ranking). That is, whenever the Lorenz ranking states that inequality has gone up, these two measures do not disagree. However, it is possible for these measures (and others) to disagree when Lorenz curves *do* cross: we provided a numerical example of this, as well as real-life instances drawing on studies of Latin American inequality.

Thus the theory of inequality measurement serves a double role. It tells us the ethical principles that are widely accepted and that we can use to rank different distributions of income or wealth, but it also warns us that such principles are incomplete, so we should not treat the behavior of any one complete measure at face value. We may not have direct information regarding the underlying Lorenz curves, but it is a good idea to look at the behavior of more than one measure before making a provisional judgment about the direction of change in inequality (if any such judgment can be made at all).

Exercises

■ (1) Connect and contrast the following concepts: (a) inequality of current income versus inequality of lifetime income, (b) functional versus personal income distribution, (c) efficiency versus equity, (d) inequality of income versus inequality of opportunities, and (e) wage inequality versus income inequality. In each case, make sure you understand each of the concepts and how they are related to each other.

■ (2) The economy of ShortLife has two kinds of jobs, which are the only sources of income for the people. One kind of job pays $200, the other pays $100. Individuals in this economy live for two years. In each year, only half the population can manage to get the high-paying job. The other half has to be content with the low-paying one. At the end of each year, everybody is fired from existing positions, and those people assigned to the high-paying job next year are chosen randomly. This means that at any date, each person, irrespective of past earnings, has probability 1/2 of being selected for the high-paying job.

(a) Calculate the Gini coefficient based on people's incomes in any one particular period and show that it suggests a good deal of inequality. Now calculate each person's average per period *lifetime* income and compute the Gini coefficient based on *these* incomes. Does the latter measure suggest more or less inequality? Explain why.

(b) Now change the scenario somewhat. Suppose that a person holding a job of one type has probability 3/4 of having the same kind of job next year. Calculate the expected lifetime income (per year average) of a person who currently has a high-paying job, and do the same for a person with a low-paying job. Compute the Gini coefficient based on these expected per period incomes and compare it with the measure obtained in case (a). Explain the difference you observe.

(c) Generalize this idea by assuming that with probability p you hold your current job, and with probability $1 - p$ you change it. Find a formula for inequality as measured by the Gini coefficient for each p, examine how it changes with p, and explain your answer intuitively.

■ (3) Draw Lorenz curves and calculate the Gini coefficient and the coefficient of variation for the income distributions (a)–(f). In each situation, the first set of numbers represents the various incomes, whereas the second set of numbers represents the number of people earning each of these incomes:

(a) $(100, 200, 300, 400)$; $(50, 25, 75, 25)$

(b) $(200, 400, 600, 800)$; $(50, 25, 75, 25)$

(c) $(200, 400, 600, 800)$; $(100, 50, 150, 50)$

(d) $(200, 400, 600, 800)$; $(125, 25, 125, 50)$

(e) $(100, 200, 300, 400)$; $(50, 15, 95, 15)$

(f) $(100, 200, 300, 400)$, $(50, 35, 55, 35)$.

[Try to understand the implicit transfers that move you from one income distribution to the other (except for the first three, which should turn out to have the same inequality — why?).]

■ (4) What are the ethical principles that we used in our measurement of inequality? Show that these principles are exactly summarized in the concept of the Lorenz curve. Argue that if there are two income distributions for which the Lorenz curves do not cross, then the Gini coefficient and the coefficient of variation cannot disagree with each other when measuring the inequality of these two distributions.

■ (5) In a world in which there are fixed minimum needs for survival, show that an application of the relative income principle runs into problems. How would you try and modify the principle to circumvent this problem?

■ (6) Suppose that there are n people in society, arranged (without loss of generality) in increasing order of income earned. Let $x = (x_1, \ldots, x_n)$ and

$y = (y_1, \ldots, y_n)$ be two income distributions (with *total* incomes the same in the two cases).

(a) Show that the Lorenz curve for x must lie inside the Lorenz curve for y if (and only if)

$$\sum_{i=1}^{k} x_i \geq \sum_{i=1}^{k} y_i$$

for all k, with strict inequality for some k.

(b) (Extra credit.) Now suppose that the condition in part (a) does hold. Show that y can be attained from x by a sequence of regressive transfers. For details, see Fields and Fei [1978].

■ (7) The Dalton transfer principle may not be a good way to judge increases in *polarization* (for a definition, see Esteban and Ray [1994] and Wolfson [1994]). To see this, begin with a society in which incomes take all values in \$100 increments between \$100 and \$1000, and in which an equal proportion of the population (1/10) occupies each of these classes. Show this income distribution in a diagram with incomes on the horizontal axis and population proportions on the vertical axis. Now draw another diagram with half the population at the income level \$250, and another half at income level \$750. Intuitively rank these two income distributions: which one has more scope for social unrest, which one might display a greater awareness of inequality, and so on.

Now show that the second distribution can be obtained by a sequence of *progressive* Dalton transfers from the first. Do you feel that your intuition is in line with the transfer principle, in this example?

■ (8) The economy of Nintendo has ten people. Three of them live in the modern sector of Nintendo and earn \$2000 per month. The rest live in the traditional sector and earn only \$1000 per month. One day, two new modern sector jobs open up and two people from the traditional sector move to the modern sector.

(a) Show that the Lorenz curves of the income distributions before and after must cross. Do this in two ways: (i) by graphing the Lorenz curves and (ii) by first expressing both income distributions as divisions of a cake of size 1, and then showing that the two distributions are linked by "opposing" Dalton transfers.

(b) Calculate the Gini coefficients and the coefficients of variation of the two distributions.

■ (9) Are the following statements true, false, or uncertain? In each case, back up your answer with a brief, but precise explanation.

(a) The Kuznets ratios satisfy the Dalton transfer principle.

(b) If the Lorenz curve of two situations do not cross, the Gini coefficient and the coefficient of variation cannot disagree.

(c) If a relatively poor person loses income to a relatively rich person, the mean absolute deviation *must* rise.

(d) The Lorenz curve must *necessarily* lie in the lower triangle of the diagram, bounded by the 45° line at the top and the axes at the bottom.

(e) The ethical principles of inequality measurement — anonymity, population, relative income, and transfers — are enough to compare any two income distributions in terms of relative inequality.

(f) If everybody's income increases by a constant dollar amount, inequality *must* fall.